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A BRIEF SURVEY OF SUPERSONICS*

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The field of acoustic research has been vastly enlarged in recent years by the fruitful application of the studies of Rayleigh and other pioneers to problems of transmission of speech and music, of their recording and reproduction, and of the effect of the surroundings of both the source of the original sound and the listener. The object of such work has been the elimination of distortion and the reproduction at the listener's ear of sound waves identical in structure with those given out at the source. In this work it has been natural that interest should chiefly be in the region of acoustic frequencies to which the human ear is sensible. However, an elastic fluid, such as air, the usual medium of sound waves, is capable of transmitting compressional vibrations of frequency many octaves above those in the audible range. The knowledge of the means of producing such waves and of their application to scientific and technological uses is of relatively recent growth and forms a most absorbingly interesting chapter of twentieth century science and engineering.

Although, prior to the World War, several investigators had succeeded, by such means as the electric spark and the singing arc, in pro-

* The Editorial Board invited Professor Hubbard to write this article, for the purpose of acquainting the Journal readers with the rapid developments in this interesting subject. The following paper by Dr. Bourgin on "Quasi-standing Waves in a Dispersive Gas" sets forth a theoretical discussion on one phase of supersonics.

ducing and studying in a limited way, wave trains of very high frequency, it was really Langevin, who, by his work in under-water signalling, provided for the first time a dependable source of supersonic waves of controllable frequency and intensity, and thereby pointed out the trail which has since led into fields of most surprising variety, interest and practical importance to the engineer and scientist.

Langevin, in his experiments, made use of properties of quartz discovered in 1880 and 1881 by Pierre and Jacques Curie and by Lippmann.[1] These properties, namely the piezoelectric effects, were found in a variety of crystalline substances. If, to consider a special case, a slab of crystalline quartz is cut so that two of the parallel faces of the slab are parallel to the optic axis of the crystal and perpendicular to an "electric" axis, the slab is found to show two effects, one the converse of the other. If the slab is put under stress it becomes electrically polarized, and, conversely, if the slab is placed in a strong electric field it suffers a change of dimensions. More specifically, if the slab is provided with electrodes on the two faces parallel to the optic axis, then any alteration of stress between the electrodes will cause a corresponding alteration of electric charge upon the electrodes, and conversely, any alteration of charge upon the electrodes will produce a corresponding alteration in the thickness of the slab. We thus have in one and the same apparatus an electrical transmitter and receiver. Langevin[2] made a mosaic of pieces of quartz, all cut in the same way, forming a large disk. The faces of this disk were covered with thin sheet metal to serve as electrodes and the whole housed under water so that one of the electrodes was in contact with the water. A generator of electrical oscillations was then connected to the electrodes, causing intense vibration in the mosaic disk, so as to generate compressional waves of great intensity in the water. These waves, being of very short length, owing to the high frequency of the oscillator, produced a supersonic beam of relatively small spread, and accordingly provided a means of directive signalling which was of use during the war, and which since, in the hands of R. W. Boyle and C. D. Reid,[3] has been used for the location of icebergs and submerged reefs. Professor Boyle[4] and his collaborators in Canada have

[1] P. and J. Curie, Comptes Rendus **91,** 294 (1880); **92,** 186 (1881). G. Lippmann, Ann. de Physique et de Chimie **24,** 145 (1881).

[2] P. Langevin, French Patent, No. 505,903, (1918); British Patent, No. 145,691, (1920).

[3] R. W. Boyle and C. D. Reid, Trans. Roy. Soc. Canada, 3rd series, **20,** section III, 233 (1926).

[4] R. W. Boyle and J. F. Lehmann, Trans. Roy. Soc. Canada. **19,** 159 (1925). R. W. Boyle and G. B. Taylor, Trans. Roy. Soc. Canada. **19,** 197 (1925).

used a method similar to that outlined for producing supersonic beams with which they have made a most comprehensive study of the properties of these waves, their velocity, reflection, refraction, interference and diffraction in water and other liquids. They showed that there was no detectable change of velocity in liquids within wide frequency limits.[5]

A most significant advance was made in 1922 when W. G. Cady[6] announced the results of his studies of the piezoelectric resonator. He showed that a piezoelectric plate provided with electrodes could be included in an electric circuit in such a way as to permit of exact measurement of its electrical and mechanical properties. He demonstrated that such a plate possessed a decrement far smaller than that of any ordinary oscillating electrical circuit. Of tremendous practical importance was his application of these principles to the development[7] of a vacuum tube circuit containing a quartz plate which was capable of maintaining electrical oscillations of a remarkably constant frequency, owing to the very small temperature coefficient of quartz and to its permanence as a mechanical system. This discovery of Cady's is the basis of the well-known stabilization of frequency of radio broadcast stations. G. W. Pierce[8] applied it to the precision calibration of electric wave meters, and W. A. Marrison[9] at the Bell Telephone Laboratories has used it as the basis of his electrical clock. Marrison found the secret of cutting rings of quartz of such dimensions as to have no significant temperature coefficient of expansion, and by the use of multivibrators the high frequency of the quartz oscillator is subdivided until a current of low frequency is obtained with which an electric clock may be run. This clock is by far the most accurate in existence. Signals sent by it to the Loomis Laboratory at Tuxedo Park, New York, have made it possible for Mr. A. L. Loomis to secure records of such precision of the rates of the best available pendulum clocks that their analysis by Brown and Brouwer has demonstrated for the first time the semidiurnal effect on pendulums of the gravity of the moon.[10]

D. W. Dye,[11] in England, in his exhaustive and fruitful study of the

[5] R. W. Boyle and G. B. Taylor, Trans. Roy. Soc. Canada, **21**, 79 (1927).

[6] W. G. Cady, Proc. I.R.E. **10**, 83 (1922).

[7] W. G. Cady, U. S. Patent, No. 1,472,583, (1923).

[8] G. W. Pierce, Proc. Am. Acad. Arts and Sci. **59**, 81 (1923).

[9] W. A. Marrison, Bell System Tech. Jour. **8**, 493 (1929); Proc. Nat. Acad. Sci. **16**, 496 (1930).

[10] A. L. Loomis, Roy. Astron. Soc. M.N. **91**, 569 (1931). E. W. Brown and D. Brouwer, Roy. Astron. Soc. M.N. **91**, 575 (1931). See also the interesting comments of C. V. Boys, Nature **128**, 651 (1931).

[11] D. W. Dye, Proc. Phys. Soc. Lond. **38**, 399 (1926).

piezoelectric resonator, and independently, K. S. van Dyke,[12] in this country, found that a quartz crystal plate provided with electrodes could be represented as an electric network containing resistance, inductance and capacity. Its decrement, as Cady had discovered, is extraordinarily small as compared with any real electric circuits. It, therefore, has unique advantages when used in combination with electric circuits, for example in the construction of electrical filters of extremely narrow band. In this and other uses the possible industrial applications of the piezoelectric effect seem almost without limit, although up to the present their importance, outside the circle of electrical communications engineers, has hardly been realized.

Another and significant direction was given to research in supersonics by R. W. Wood and A. L. Loomis[13] who made a comprehensive study of the physical and biological effects of supersonic waves. Their apparatus consisted of a two inch disk of quartz resting upon a lead plate on the bottom of a shallow dish filled with transformer oil. The upper surface of the quartz was covered by a thin foil of metal which was grounded. The foil and the lead plate serving as electrodes, were connected to the secondary of a 2 kw generator of electric oscillations. By this means upward of 50,000 effective volts of oscillating potential were applied to the disk, producing many astonishing effects. When the quartz was excited near its resonant frequency, a mound of oil was raised several centimeters above the oil level in the dish and was in most violent agitation. A thermometer immersed in the oil showed but a moderate increase in temperature, but if the finger of an observer were touched to the oil for an instant a most painful scalding sensation was felt. If a test tube containing paraffin and water or mercury and water was held in the oil bath, a rapid dispersion of the paraffin or mercury in the water took place, yielding a suspension of unusual permanence. The cells of animal or vegetable tissues immersed in a bath in contact with the oil were disrupted. A glass rod drawn down at one end to a fine point was found capable of conducting energy from the oil bath to such a degree that the point was heated to redness and a bead of glass formed.

These and many similar observations have led to a great variety of studies by biologists, chemists and others. The vibrations of small intensity are of possible therapeutic value, and in great intensities can undoubtedly be developed for the preparation of fine suspensions or for the extraction of juices from plant cells. Studies recently made appar-

[12] K. S. van Dyke, Proc. I.R.E. **16**, 742 (1928).

[13] R. W. Wood and A. L. Loomis, Phil. Mag. **4**, 417 (1927).

ently show the practicability of using supersonic radiation for the destruction of bacteria, as in the sterilization of milk.

Impetus to research was given in a quite new direction by G. W. Pierce[14] who utilized a vacuum tube circuit containing a piezoelectric plate for the measurement of supersonic velocities in air and in CO_2. The quartz plate under these circumstances radiates compressional waves into the surrounding gas. If a reflector be placed opposite and parallel to one of the vibrating faces of the plate the waves are reflected back to the quartz. As the reflector is moved toward or away from the plate, positions of resonance in the gas are found, and as the reflector passes through positions of resonance remarkably sharp changes occur in the plate current of the vacuum tube. Acoustically, this experiment is analogous to the well-known Kundt's tube experiment. The positions of resonance, measured by a micrometer screw, are an acoustic half wave-length apart, and, the frequency of the oscillating system being easily measured, the velocity can be determined at once. The figures yielded are of extraordinarily high precision, and enabled Pierce to detect a variation of velocity with frequency in CO_2, long looked for in vain. These experiments of Pierce have led, as we shall see, to a variety of studies of great interest to physicists and chemists. Because of the extreme shortness of the ultrasonic wave-lengths, a crystal of moderate dimensions radiates plane waves for some distance from its face. The behavior of the apparatus of Pierce is thus analogous in many ways to that of an optical interferometer. For this reason it is customary to speak of Pierce's apparatus as an acoustic interferometer.

An interferometer of the sort just described cannot be used for the acoustic study of liquids for the reason that a piezoelectric plate used as an oscillator will not vibrate if its surfaces are contaminated by moisture. However, the experiments of Langevin, of Boyle, and of Wood and Loomis had shown that by driving a crystal by an electric generator of oscillations a beam of supersonic radiation could be made to pass into a liquid. Accordingly, Mr. A. L. Loomis and the writer[15] devised a supersonic interferometer for liquids by using a thin metal membrane in contact with a driven piezoelectric plate as the source of waves, and a piston moved by a micrometer screw as a reflector. Reflector positions at resonance are found by observing the sharp reaction produced in the electrical current in the oscillator circuit as the piston

[14] G. W. Pierce, Proc. Am. Acad. **60**, 269 (1925).

[15] J. C. Hubbard and A. L. Loomis, Phil. Mag. **5**, 1178 (1928); A. L. Loomis and J. C. Hubbard, J.O.S.A. and R.S.I. **17**, 295 (1928).

passes through successive half-wave distances. The precision with which supersonic velocities in liquids can be determined was found to be of the same high order as that of determination of velocity in gases by Pierce. The source is chosen so as to present to the liquid a plane surface of large diameter as compared with the wave-length in the liquid, and the reflector distances from the source are never greater than the diameter from the source. By these means the wave system is always plane, diffraction effects are negligible and the troublesome tube corrections, which must be made at low frequencies, or when a long path is employed, are entirely avoided. It was shown that the development of this apparatus makes practicable the study of liquids in very small amounts, a desirable feature, because of ease of temperature regulation and the rarity of many compounds. It can be used conveniently, similarly to the refractometer or saccharimeter, for analysis and the detection of impurity or for the control of composition in chemical manufacture. It was also pointed out that the supersonic interferometer for liquids may be used as a secondary standard of precision for the determination of the frequency of electrical oscillations and for the calibration of wave meters. This proposal gains in force from the recent precise determination, using the interferometer for liquids, of supersonic velocity in distilled water by C. R. Randall[16] at the Bureau of Standards.

Of remarkable interest and of probably great importance are the experiments of Debye and Sears[17] on the scattering of light by supersonic waves. A quartz plate with silvered (electrode) faces is immersed in a glass trough of rectangular cross section filled with an insulating liquid so that when the electrodes are connected with a radio-frequency oscillator a train of supersonic waves travels down the trough. At right angles to the trough is arranged a slit and lens so that a parallel beam of monochromatic light may be sent through the liquid at right angles to the path of the supersonic waves. After passage through the trough the light is gathered by a telescope or a camera, when it appears that instead of a single image of the slit there is a series of its diffraction images. These images are formed in somewhat the same way by visible light traversing a medium of equally spaced density variations as diffraction images are formed by x-rays reflected from the regularly spaced molecules of a crystal. From the positions of the images, the wave-length of the light and the frequency of oscillations, determined

[16] C. R. Randall, Bur. Stand. Jour. Res. **8**, 79 (1932).
[17] P. Debye and F. W. Sears, Proc. Nat. Acad. Sci. **18**, 410 (1932).

by a wave meter, the supersonic velocity in the liquid is measured. With only slight modification this experiment could be applied to conducting liquids, and without doubt can be developed to a high degree of accuracy. The authors report that they have observed no change in velocity with a tenfold variation of frequency, the latter having been carried to 16.5×10^6 cycles, by far the highest frequency for which supersonic waves have been measured. The practical advantages, aside from the great theoretical interest, of these experiments, can be seen at once from the fact that the space containing the liquid is closed, the measurements requiring no moving part such as a piston.

Thus far we have considered only the piezoelectric properties of quartz as the effective agent in the production of supersonic waves. The magnetostriction oscillator has been developed for the same purposes by G. W. Pierce.[18] It is particularly valuable in the lower regions of supersonic frequency, being available from the audible range up to that easily accessible with quartz.

During the past few years physicists and chemists have become greatly interested in the study of the specific heats of gases, particularly with respect to the relative parts played by the vibrational and rotational frequencies of molecules at different temperatures. An important approach to this problem lies in the study of velocity and absorption of sound in gases at different frequencies and at higher temperatures. To this end the writer began in 1926 to examine available methods. The Pierce interferometer gives the velocities and has been much used for the determination of the ratio of specific heats, but owing to the fact that the amplitudes of its readings involve the characteristics of the vacuum tube and its circuit, it did not seem practicable for the measurement of absorptions.

Accordingly, the interferometer for liquids was adapted to the study of gases.[19] In the arrangement adopted the interferometer was included in a resonant electric circuit loosely coupled to a high-frequency generator of oscillations of constant frequency and amplitude. This arrangement permitted an electrical analysis of the interferometer to be made by the methods developed by Cady and extended by Dye for the study of the quartz resonator itself. In order to make the analysis complete, it was necessary to work out the theory of the effect of a fluid column bounded at one end by a vibrator and at the other by a reflector, and to determine the modification produced by such a column in the equiva-

[18] G. W. Pierce, Proc. Am. Acad. Arts and Sci. **63**, 1 (1928).
[19] J. C. Hubbard, Phys. Rev. **35**, 1442 (1930); **36**, 1668 (1930).

lent electric network of the system.[20] This accomplished, it was possible to develop a method of measuring both the velocity and absorption of supersonic waves in a gas in terms of galvanometer readings, lengths of column and frequency of vibration.[21]

In the meantime H. O. Kneser[22] has applied the method of Pierce to the measurement of the supersonic velocity in several gases, particularly CO_2, and has found between the frequencies 100 kc and 600 kc, a marked increase in the velocity amounting to 3.7 percent. Moreover, E. Grossmann,[23] by a direct method, using the radiation from a small opening placed in front of a vibrating quartz, so as to approach a pinhole in effect, and receiving the radiation on a second quartz plate, used as a detector of the radiation, which could be placed at different distances from the source, was able to measure the absorption in CO_2. Obviously, if there were no absorption, the intensity of radiation would be inversely as the square of the distance of the detector from the source, correcting of course for the solid angle subtended by the detector and the phase relations of the waves reaching its various parts. The defect in the inverse square law allows the absorption to be computed. Grossmann finds a marked absorption in the region of 100 kc.

In the classical theory of propagation of sound we have first the equation of Newton in which the velocity of sound is expressed in terms of the pressure and density of the gas. This theory as will be remembered, gives values of velocity much lower than those actually found. LaPlace corrected Newton's equation by including the ratio of specific heats of the gas, the compressions and rarefactions in a sound wave being considered as adiabatic. This correction gave remarkably close agreement between theory and experiment, so much so that sound velocity measurements have for a long time been used for determining the ratios of specific heats of gases. It was recognized by Stokes that the inner friction in a gas would produce absorption at high frequencies, and Kirchhoff showed that heat conductivity would likewise have an effect. Herzfeld and Rice[24] were the first to study the effect of these factors on the velocity of sound. In addition they pointed out that the slowness of energy exchange between the translational and internal degrees of freedom of gas molecules should also affect velocity,

[20] J. C. Hubbard, Phys. Rev. **38**, 1011 (1931).
[21] J. C. Hubbard, Phys. Rev. **41**, 523 (1932).
[22] H. O. Kneser, Ann. d. Physik **11**, 779 (1931).
[23] E. Grossmann, Ann. d. Physik **13**, 681 (1932).
[24] K. F. Herzfeld and F. O. Rice, Phys. Rev. **31**, 691 (1928).

and they developed a theory of dispersion and absorption which they carried to a first approximation. A theoretical study of the effect of the rate of energy exchange has also been made by Bourgin,[25] and by Kneser,[26] who finds that his experimental results for the velocity in CO_2 can be explained on the assumption that there is an exchange between the translational and vibrational degrees of freedom of the CO_2 molecule involving a time of about 10^{-5} seconds.[27] Roughly, one may picture a gas composed of complex molecules as being stiffer for vibrations above a certain frequency range than for those below. If the vibrations are slow enough, all the degrees of freedom take their full share of energy. As the frequency increases, the vibrational degrees of freedom for which there is a relatively slow rate of energy exchange will fail to take up their share of heat and the apparent ratio of specific heats, and therefore the velocity, will be increased.

From the foregoing it will be clear that supersonic research is entering upon a phase in which it can be expected to be of service to the molecular physicist. He relies ordinarily upon light or intense electric fields to produce disturbances of equilibrium which he can measure. In supersonic waves he has a new agent, a mechanical one, with which to work.

In a broad survey such as the foregoing, it is manifestly impossible to portray the great variety of detail which has been developed in supersonic research, and only a few of the most important problems and applications have been mentioned. Enough has been given, however, to show that while supersonics is a logical extension of acoustics at audible frequencies, the fact that we are in a new frequency region and one which may be more closely related to periods of time involved in molecular physics gives us assurance that new aspects of nature will be brought into view.

October 10, 1932

[25] D. G. Bourgin, Phil. Mag. **7**, 821 (1929); Phys. Rev. **34**, 521 (1929).
[26] H. O. Kneser, Ann. d. Physik **11**, 761 (1931).
[27] Henry, Nature, Feb. 6 (1932).

QUASI-STANDING WAVES IN A DISPERSIVE GAS

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Abstract

For application to standing wave experiments an explicit formula is derived for the influence of absorption on the node position for quasi-standing waves. The effect is of the "second order" in the coefficient of absorption and is inversely proportional to the frequency.

For high-frequency sound work the only method thus far found applicable for velocity and absorption measurements has been one involving a piezoelectric oscillator which is used in some modification of a standing wave experiment.[1] The nodes are located by back reaction on the oscillating crystal and it is the purpose of this paper to provide the theoretical validation of such a procedure. It will be assumed that the change of type of the waves is of negligible effect.

The question presented is a simple boundary value problem associated with the differential equation system governing the propagation of sound in a dispersive gas. Such a system has been treated in two recent papers of the writer.[2] For instance, in the interest of simplicity, it will be assumed that the reflector is perfectly rigid and that the source executes uniform vibrations of negligible amplitude. Taking the mass ve-

[1] For a bibliography on this subject cf. the paper by J. C. Hubbard in this issue.

[2] D. G. Bourgin, Nature **122**, 133 (1928); Phil. Mag. **7**, 821 (1929); Phys. Rev. **34**, 521 (1929). (These last will be referred to as I and II, respectively.)

The writer takes this opportunity to point out that there are a number of misprints occurring in I and II mostly concerned with the appearance of N. These are in the main obvious since the final formulae are correct. In the definition of B_s the subscripts rs inside the summation should be interchanged and "per molecule" should replace "per collision" in the definition of B, p. 828.

More serious is the error in Eq. (25) of I and the corresponding generalized Eq. (1.2) of II. This latter should be

$$\frac{\partial}{\partial t}\sum_w N_w\omega_w = -\frac{\partial}{\partial x}(\tfrac{2}{3}NK) + \cdots$$

instead of

$$\frac{\partial}{\partial t}N\bar{v} = -\frac{\partial}{\partial x}(\tfrac{2}{3}SK) + \cdots.$$

It may readily be verified that to the order of approximations occurring in II the final expressions are correct if S/N be given the new interpretation $N/\sum_w N_w m_w$, i.e., the reciprocal mean value of the mass instead of the mean value of the reciprocal mass. Hence on p. 524 the word reciprocal in paragraph 2 as well as the whole first footnote on that page are to be deleted. Instead of Eq. (6), II, the formula for the sound velocity in air is

$$V_{12} = [V_2{}^2 5m_2/(4m_1 + m_2)]^{1/2} \sim 337 \text{ m/sec.}$$

locity $v(x, t)$ as the dependent variable the boundary conditions are

$$v\,|_{0} = Iv_0 e^{i\nu t}, \quad v\,|_{l} = 0, \tag{1.00}$$

Actually, of course, the boundary conditions are functionals in $v(x, t)$ and the rigorous solution is to be gotten by a method of successive approximations. From this point of view the present development supplies the first approximation and one which is expected to be adequate for most interpretations.

For small amplitudes the differential equations are satisfied by $e^{i(\nu t \pm x/\lambda)}$ where ν is the frequency and the wave-length λ is complex. A solution satisfying the boundary conditions is built up in the form

$$v = I\{A_1 e^{i(\nu t + x/\lambda)} + A_2 e^{i(\nu t - x/\lambda)}\}. \tag{1.01}$$

The boundary conditions are sufficient to determine A_1 and A_2 and we easily derive

$$v = I\,\frac{v_0[e^{i(\nu t+(x-l)/\lambda)} - e^{i(\nu t-(x-l)/\lambda)}]}{e^{-il/\lambda} - e^{il/\lambda}}. \tag{1.011}$$

We have

$$(\partial/\partial t)(mNv) = -(2/3)KN \tag{1.02}$$

where m and N are the mass of a molecule, N is the number of molecules per cc and K is the kinetic energy per mol. On interpreting KN as p and neglecting $v(\partial N/\partial t)$

$$(\partial/\partial x)p \sim \partial v/\partial t \tag{1.021}$$

except for an unimportant factor. On integrating this equation with the value of v given by Eq. (1.001) and omitting factors not containing t, x or l we obtain

$$\delta p \sim I\,\frac{e^{i(\nu t+(x-l)/\lambda)} + e^{(i\nu t-(x-l)/\lambda)}}{e^{-il/\lambda} - e^{il/\lambda}} \tag{1.03}$$

where δp is the variable part of p.

For application to the experimental method[3] we assume that the phase of δp is of negligible importance[4] in determining the reaction on the

[3] Pielemeier has taken up this question by following out the successive reflections of a *pressure* wave, Phys. Rev. **34**, 1184 (1929). The writer has not analyzed Pielemeier's treatment but it does not seem to be altogether sound as it stands for he finds the position of the extrema to be strictly independent of the absorption in contradiction with the conclusion quantitatively formulated in Eq. (1.04).

[4] This assumption is made because it yields an upper gauge of the error arising from neglect of damping. The probably more accurate condition that only the "in phase" component, the first parenthesis in Eq. (1.032), is of importance in the back reaction does not affect the direc-

oscillating crystal. We are, therefore, interested in the extrema of $\delta p(0)$ for varying l.

If there is absorption there are, evidently, no absolute nodes but only positions of least δp amplitudes and the term anti-node must be qualified also. Since the amplitude of a product of complex quantities is the product of the respective amplitudes and the time exponential is of modulus 1 there results

$$|\delta p(0) = C\left|\frac{e^{l/j\lambda} + e^{-l/j\lambda}}{e^{l/j\lambda} - e^{-l/j\lambda}}\right| = C\left|\frac{(e^{l/j\lambda} + e^{-l/j\lambda})(e^{l/\bar{j}\lambda} - e^{-l/\bar{j}\lambda})}{(e^{l/j\lambda} - e^{-l/j\lambda})(e^{l/\bar{j}\lambda} - e^{-l/\bar{j}\lambda})}\right| \quad (1.031)$$

where the bar denotes the conjugate complex quantity and C is a constant. Let us write $1/j\lambda = a + (\nu/V)^i$ where V is the real part of the velocity and a measures the absorption. Thus

$$\begin{aligned} |\delta p|^2 &= C^2\left\{\frac{(e^{2al} - e^{-2al})^2 + 4\sin^2 2\nu l/V}{(e^{2al} + e^{-2al} - 2\cos 2\nu l/V)^2}\right\} \\ &= C^2\left\{\frac{e^{2al} + e^{-2al} + 2\cos 2\nu l/V}{e^{2al} + e^{-2al} - 2\cos 2\nu l/V}\right\}. \end{aligned} \quad (1.032)$$

From Eq. (1.032) the condition for extrema is found to be

$$\tan 2\nu l/V = -(aV \tanh 2al)\nu. \quad (1.04)$$

a and v are assumed known, and successive values of l are determined from the experimental condition of reaction on the crystal. Hence V may be determined. The right side represents a correction term, heretofore disregarded, to the simple standing wave calculations. The nature of the correction is fairly obvious. Suppose l_n the position of the n^{th} extremum (really only the extrema corresponding to even n values are accurately located). Then on the simple theory we should write

$$2\nu l_n/V_0 = n\pi. \quad (1.05)$$

From the fact that the right side of Eq. (1.04) is negative, it follows that the corrected value of Eq. (1.05) according to the analysis developed is

$$2\nu l_n/V = n\pi - |\delta_n|. \quad (1.051)$$

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tion or implications of the error but reduces it to one-fourth that determined from Eq. (1.04). More explicitly the right side of Eq. (1.04) is replaced by

$$-aV(\coth 2al - \operatorname{sech} 2al \sec 2\nu l/V)/2\nu$$

which is about

$$-aV(\sinh 2al/\cosh 2al + 1)/2\nu \frown -aV(\tanh 2al)/4\nu.$$

The simple theory is seen to yield calculated velocities which are too low. δ_n is calculated to an adequate approximation by writing $2\nu l_n/V = n\pi$ on the right side of Eq. (1.04) and solving. The error in V introduced by using Eq. (1.05) is practically $(aV_0^2/n\pi\nu)$ tanh $2al$. Its variation with n is gotten from

$$(\partial/\partial l)(\tanh 2al/l) \sim (2al \operatorname{sech}^2 2al - \tanh 2al)/l^2 < 0.$$

Thus the error is a decreasing function of l (or n) so that the uncorrected velocity computation will indicate a *falling* velocity as the reflector is moved toward the source,[5] (i.e., decreasing l). The correction is practically negligible for the usual non-absorptive gases experimented upon at supersonic frequencies.

Sufficient data[6] exist for CO_2 to compute the correction demanded by this paper. The following numerical values are used for a first calculation

$$\nu = 6\times10^5, \quad a \sim 1, \quad \nu/V \sim 20.$$

Then

$$\delta_1/\pi \sim 1/400, \quad \delta_5/5\pi \sim 1/470.$$

The error in V^2 varies therefore from 0.5 percent for $n=1$ to about 0.4 percent for $n=5$. Because of the strong absorption this is the order of magnitude of the experimentally feasible values for n (or l_n). While this is much greater than the experimental error in the non-absorptive region, it is too small to affect the interpretation of the large change of velocity in the intermediate region as found in Kneser's experiments.

June 11, 1932

[5] The change of velocity with distance from the source for weakly absorptive gases as observed by Reid, Phys. Rev. **35**, 814 (1930); Phys. Rev. **37**, 1147 (1931) and others is not to be explained by the considerations developed in this paper since the effect is in the opposite direction.

[6] H. O. Kneser, Ann. d. Physik **11**, 761 (1931).

ACOUSTICS OF LARGE AUDITORIUMS[1]

By SAMUEL LIFSHITZ
University of Moscow

In connection with the proposed erection of the Palace of the Soviets in Moscow, which will include two auditoriums, one to accommodate 15,000 persons, and the other 6000 persons, a number of experiments

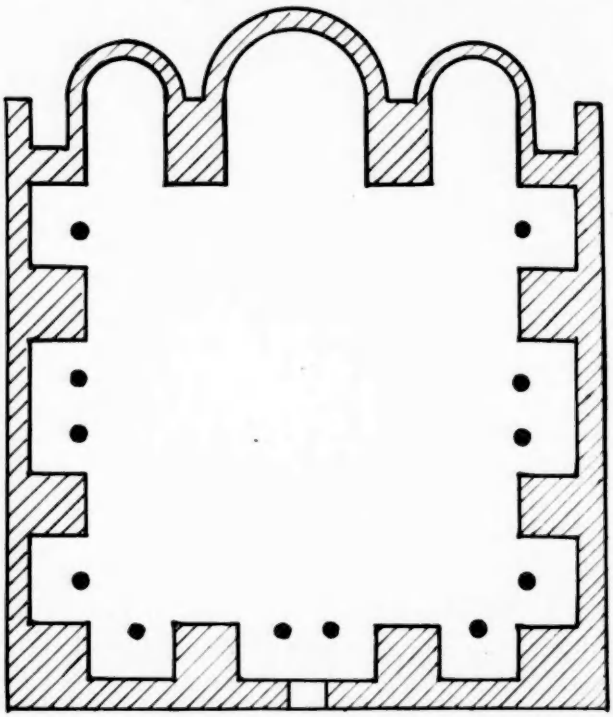

FIG. 1. *Plan of unfinished church in Moscow, used for acoustic experiments.*

were carried out to gain useful information regarding the acoustics. For investigational purposes, use was made of a partly completed building intended for a church in Moscow on the Meusa Square; the construction work of which, after erecting the walls and roof, was stopped in 1914. This empty building, of 43,000 cubic meters volume, with a simple plan, without columns, inside walls or partitions, and having no echoes or focal points, proved very advantageous for the experiments. Fig. 1 shows the plan and Fig. 2, the cross section.

[1] Original manuscript edited by F. R. Watson.

Two lines of investigation were conducted: 1. A study of the articulation field, as well as other problems of articulation. 2. A determination of the musical optimum of reverberation.

ARTICULATION FIELD

Knudsen, in conducting experiments in articulation, characterizes rooms as having an average percentage of articulation, depending on the existing conditions of the room. That is, since the percentage of articulation is not the same in different parts of a room, the differences

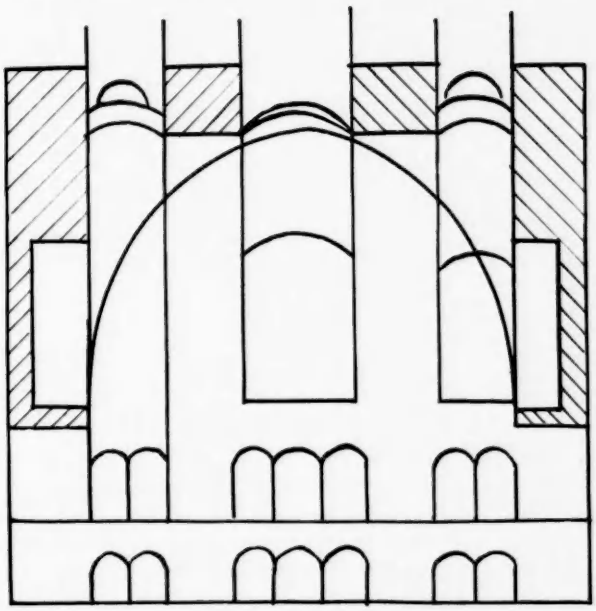

FIG. 2. *Cross section of the church.*

being greater for larger rooms, the determination of the average depends on the positions in which the measurements are made. The articulation percentage in different points of the room changes in different directions from the speaker, forming an "articulated field," or "field of articulation," as we may choose to call it. Such an articulated field was determined in the previously described building, with the results shown in Fig. 3. A speaker at *A* read articulation tables in the Russian language. It is to be noted that the percentage of articulation is distributed regularly around the speaker, the closed curves showing lines of equal value. The results give an indication of the most rational arrangement of speaker and auditors. Similar experiments were taken in the open

air (Fig. 4), and it is seen that the percentage of articulation beyond the speaker decreases more rapidly in the open air than in the closed room.

DISTRIBUTION OF ARTICULATION PERCENTAGE ABOUT A SPEAKER

The decrease in the articulation percentage with increase of distance depends on the polar direction from the speaker's mouth. Fig. 5

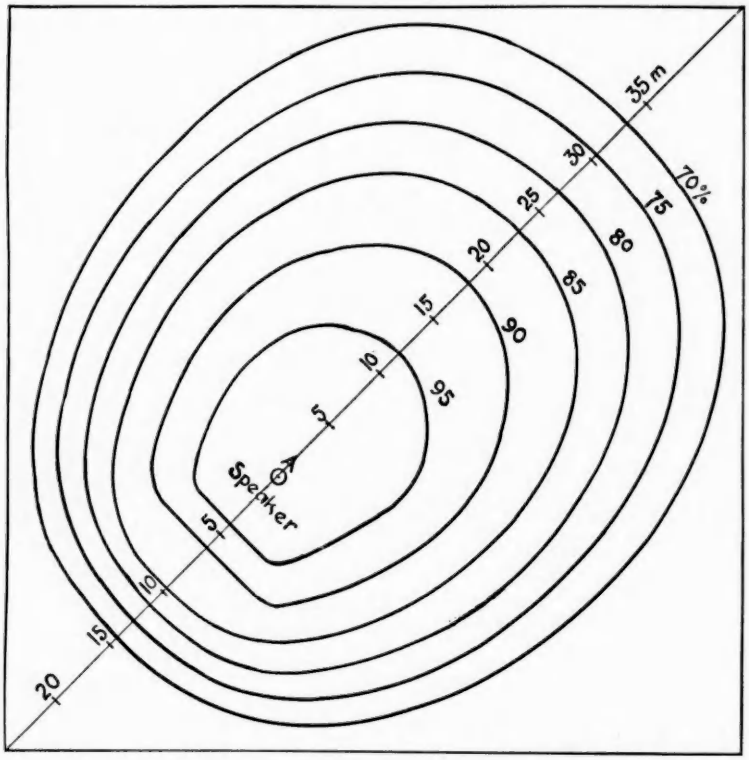

FIG. 3. *Field of articulation in a closed building, with noise level of 30 decibels, distances given in meters.*

pictures the results for the closed room, the curves indicating respectively, the distribution directly ahead of the speaker (0°), at 45°, 90°, 135° and 180° (back of speaker). During these experiments, the same speaker read the articulation tables. His voice was of average loudness and average distinctness. When speaking as usual, not thinking about the distinctness, the articulation percentage decreased, as pictured in Fig. 6.

EFFECT OF DECREASED REVERBERATION ON ARTICULATION

By varying the absorption of the room, it was possible to take measurements of the articulation for different times of reverberation. Unfortunately, the change of absorption in such a large volume required the installation of absorbing materials in large quantities (300 tn of boards and 25,000 meters of soft cloth) so that the standard time of

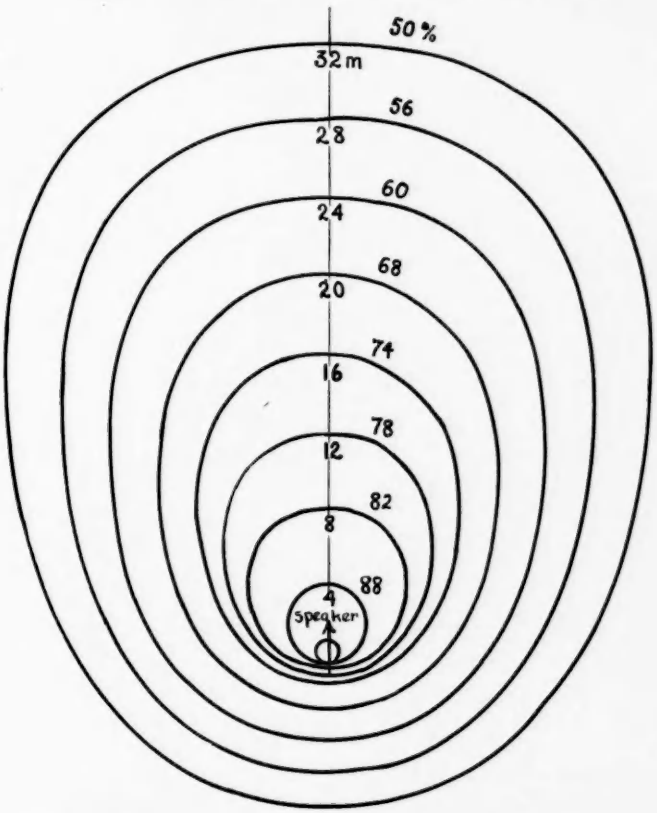

FIG. 4. *Articulation field in the open air. Winter experiments on the snow, noise level of 40 decibels, distances given in meters.*

reverberation (at 60 db loudness) could be varied only within the limits of 4 to 2 seconds. The average percentage of articulation for the entire room, as well as for different positions in the room, remained constant in this range of reverberation within the limits of error of the experiment (1–3 percent).

It took about a month's time to install the absorbing material so as to reduce the reverberation from 4 to 2 seconds. During this time, a

number of separate measurements of percent articulation were made with the help of about 30 students. The group of students changed somewhat in the individual members, but the speaker was the same throughout. The accuracy of the results obtained was such that the 3 percent

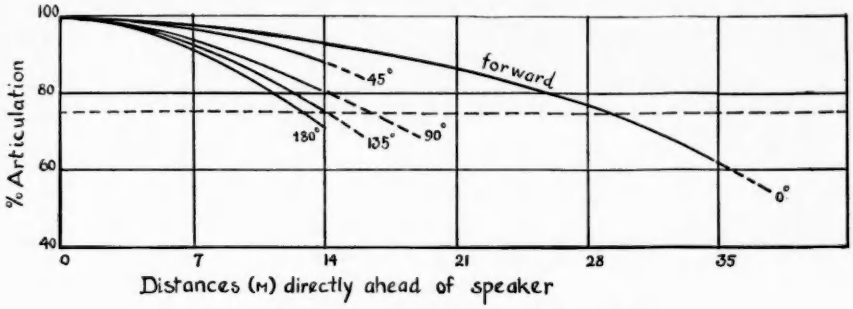

FIG. 5. *Distribution of articulation about a speaker.*

variation of percent articulation could not be determined exactly. As constant sources of error, we can assume some unconsidered variation in background noise pattern (since the measurements were taken in the winter time), also, any variation in the speaker's voice due to the changing absorption. The average percentage of articulation for the room is

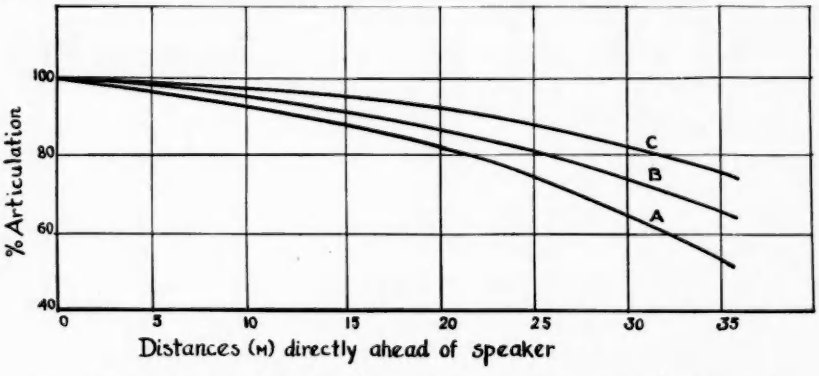

FIG. 6. *Curves of articulation directly ahead of the speaker, curve A showing usual distinctness, B, more care in speaking and C, perfect distinctness.*

shown in Fig. 7 by the solid line. Line *B* is the percentage for a point 30 meters in front of the speaker.

The reverberation time was measured experimentally in each case by two methods, one, a modification of the Olson-Kreuzer method, but with compensation for the condenser discharge; and two, Sabine's ear method for measuring the reverberation time for different intensities.

The average of the values obtained by these two methods was taken as the true value. The source of sound was a dynamic reproducer (Blathaller) in connection with a vacuum tube oscillator and a 200 watt amplifier.

RELATION BETWEEN ARTICULATION PERCENTAGE AND VALUE OF REVERBERATION

As articulation percentage undoubtedly decreases with further variation of reverberation time, we have in this case, obviously, an extremely long and sloping curve instead of a pronounced optimum. The possible decrease is shown with dotted lines in Fig. 7. This slanting increase of optimum requires further checking, since the Knudsen theo-

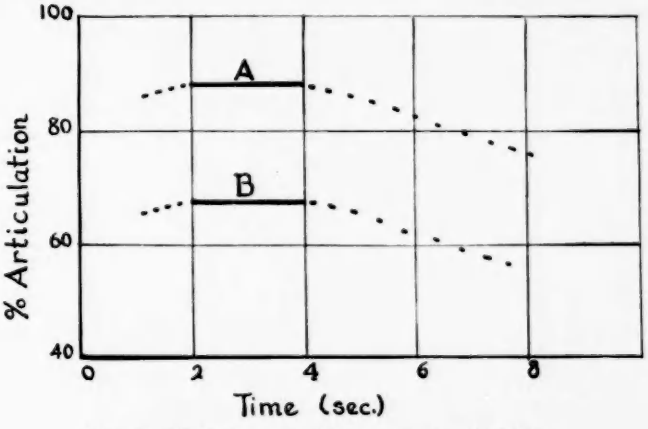

FIG. 7. *Effect of reverberation on speech articulation.*

retical curve for articulation in an audience hall of 45,000 cubic meters volume shows a greater variation of articulation with reverberation than that observed in our experiments. It is of interest to compare this slanting curve for articulation with the musical optimum which will be mentioned later and which is definitely determined with an accuracy of 0.2 sec. of reverberation time.

MUSICAL OPTIMUM OF REVERBERATION

We have defined musical optimum of reverberation[2] as the time of reverberation at which singing and the tones of musical instruments become most beautiful. For times of reverberation smaller than the optimum, singing and sounds of musical instruments become dry, but for times greater than the optimum, they become hollow. The musical

[2] S. Lifshitz, Phys. Rev. **27**, 618 (1926).

ear determines the reverberation optimum up to a 5 percent accuracy. A deviation of 10 percent from the optimum becomes noticeable for an ordinary listener.

The theoretical relation between reverberation optimum and volumes of audience halls was determined by us in 1926 in the form of a formula (Eq. (4)).[2] We took two constants as a basis, when working out this formula: The average loudness of sound in small audience halls which was determined by us as equal to 60 db with some surplus, and reverberation time in the "column" hall with 12,500 cubic meters volume which we consider perfect for musical performances. Both these constants now require some correction.

During the current year we repeated acoustical experiments in the column hall and found that the reverberation optimum for the tone 512 vibrations per second was equal to 1.7 seconds (instead of 1.75 seconds which we assumed previously). Since the average intensity of sound in small audience halls, according to published data in recent years, should be taken equal to 68 db as the most probable value (Knudsen), we can, on the basis of these new data, make corrections for our conclusions of 1926.

Eq. (1), expressing our statement that the degree of musical perception remains constant in different halls is as follows:

$$tE(\text{db}) = C. \tag{1}$$

Eq. (2) is as follows:

$$T_{\text{opt.}}^{1/2}(6n + \log_{10} a_1 - \log_{10} a_2) = 6.175n \tag{2}$$

where a_1 is the absorption of a small audience hall for which $T_{\text{opt.}} = 1.06$ sec. and a_2 is the absorption of a large hall.

As before, we take the column hall as a model from the acoustical viewpoint. We have the volume $V = 12{,}500$ cubic meters; $T_{\text{opt.}} = 1.7$ sec.; and $a_2 = 1210$.

Then after substitution, the Eq. (2) is as follows: $1.262n + \log_{10} a_1$ 3.081. If $E(\text{db}) = 68$, then $n = 1.133$, $C = 82$. Hence the following values: $a_1 = 45$, $V_1 = 290$ cubic meters. That is, V_1 is the limitation volume in which, starting with the smallest, the average level of loudness of a soloist is equal to 68 db.

Hence we obtain the equation for optimum:

$$\log_{10} V = 9.237 + \log_{10} T_{\text{opt.}} - 6.996/(T_{\text{opt.}})^{1/2}. \tag{3}$$

When developing this Eq. (3) we have assumed as before that the vocal power of a soloist for halls larger than 290 cubic meters remains con-

stant. In fact, this is not quite true. The power of a soloist in large halls increases continuously, though not so much as the increasing absorption. However, in this case, we have to take into account another process compensating this growth of power: With the increase of reverberation time in large halls, the growth of loudness becomes slower and thus for equal tempo of performance, this growth of loudness being delayed, at least partially compensates the increased power. Consequently, the supposition that the average power of a soloist remains constant gives results very close to reality.

Fig. 8 shows the curve A according to Eq. (3), while curve B is due to Békésy.[3] As can be seen in Fig. 8, the optimum of 2.04 sec. corresponds

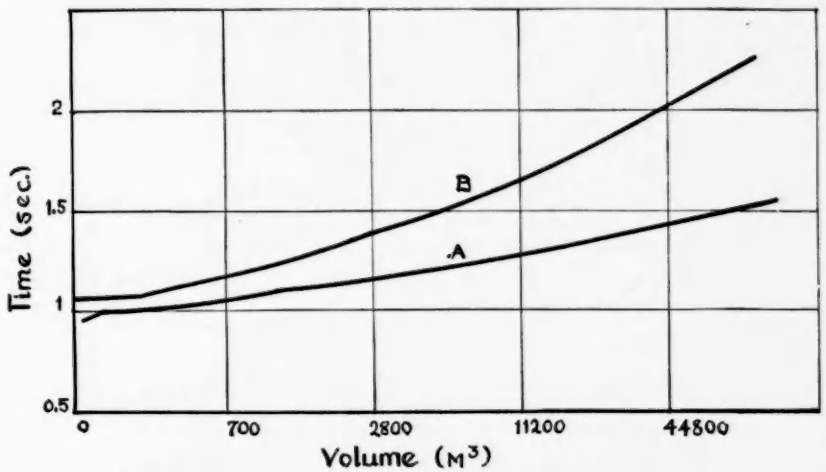

FIG. 8. *Dependence of musical optimum of reverberation upon volume.*

to a volume of 43,000 cubic meters according to Eq. (3) while according to Békésy's theory the optimum equals 1.44 sec. Thus it becomes possible to compare these two theories in an experimental way. For experimental checking we used the room of 43,000 cubic meters already mentioned in which different vocal and musical performances were listened to, with varying absorption.

Performances of the following soloists and ensembles were tested, baritone singer, soprano singer, violinist, violincellist, oboist, flutist, and a string quartet. The optimum time of reverberation for all performances was found to be within the limits 2.1 to 2.3 sec. according to the musical taste of musical experts. These values for $T_{opt.}$ approach closely

[3] I. Békésy, *Bemerkungen zur Theorie der günstigsten Nachhalldauer von Räumen*, Ann. d. Physik [5], **8**, No. 7 (1931).

the theoretical value according to the curve A, and differ considerably from values according to the curve B.

Some deviation towards larger reverberation than given by formula (3) should be attributed to the unusual size of the hall, the height of which is too large, 30 m as compared to the length and width (40×40 m).

CONCLUSIONS

On the basis of the results obtained we draw the following conclusions:

1. Distinctness of speech in large rooms can be characterized not only by the average percentage of articulation, but, which is more exact, by "articulation field."

2. Based on data of the articulation field it is possible to draw some conclusions regarding the allowable limits of sizes of rooms, assuming definite percent of articulation for the remotest places.

3. The best form of plan for large rooms is based on lines of equal percentage of articulation.

4. The curve of articulation optimum is a very slanting one.

In Fig. 9, curve A shows Knudsen's[4] theoretical curve of articulation for a hall with volume of 45,000 cubic meters, while the straight line B gives the results of our experiments, and the straight line C the results of experiments of Wayne B. Hales in the Salt Lake Tabernacle with 35,500 cubic meters volume. Absolute values of the percentage of articulation in our experiments for 2.0 sec. and Hales' experiments for 2.3 sec. are made equal for comparison with the values of Knudsen. Fig. 9 shows that experimental data demonstrate a very slanting increase of optimum. This circumstance induces us to reconsider the question as to what value of reverberation optimum should be followed in audience halls assigned both for music and speech. In such cases Knudsen recommends that the arithmetical mean between articulation and musical optimum be taken. On theoretical and practical experience, we would recommend that the reverberation time be taken 10 percent lower than the musical optimum. As an example let us take an audience hall of 43,000 cubic meters. Musical optimum of reverberation—curve A according to Fig. 8, equals 2.04 sec. The articulation optimum according to Knudsen is 1.3 sec. For halls having various purposes Knudsen recommends the optimum of 1.67 sec. while we prefer the optimum equal to 1.84 sec. At 1.67 sec. the musical optimum will decrease 22

[4] Knudsen, *The Hearing of Speech in Auditoriums*, Jour. Acous. Soc. Am. **1**, 56 (1929).

percent, which will result in a very noticeable deterioration of sonorousness of musical performance. At 1.84 sec. the musical optimum will decrease only 10 percent and the deterioration of sonorousness of musical performance will be very inconsiderable (at 5 percent difference it is generally only slightly noticeable). Meanwhile, the difference in the articulation percent for 1.67 sec. and 1.84 sec. due to the slanting increase of the curve, will be only 0.5 percent according to curve *A*, Fig.

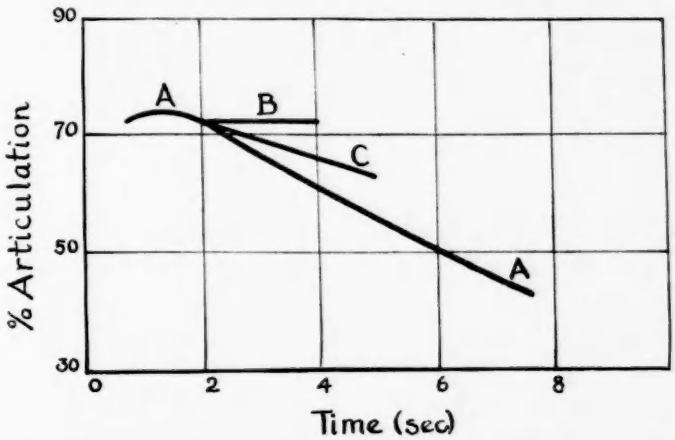

FIG. 9. *Curves of articulation optimum, A—Knudsen, B—Lifshitz, C—Hales.*

9, or will be still less according to curve *B*, i.e., it will not be perceived. The same considerations hold good for audience halls of other volumes also.

The experimental part of the work on the definition of articulations has been carried out by Mr. A. Rabinovitch, who worked up the material obtained, while the experiments concerning determination of reverberation were carried out by Mr. R. Ris, to whom we extend our sincere appreciation. We also wish to thank the Electroacoustical Laboratory of the Scientific Research Institute of Communication for their cooperation in the installation of measuring instruments, as well as the Management of Palace of Soviets Construction, in the person of Scientific Secretary Mr. D. Smolin, for his assistance in the organization of experiments.

September 24, 1932

ACOUSTIC PICK-UP FOR PHILADELPHIA ORCHESTRA BROADCASTS

By J. P. MAXFIELD
Electrical Research Products, Inc.

Successful broadcasting of artistic musical entertainment involves the cooperation of artist and engineer. In the present state of the art, the best results are being obtained where all of the artistry is controlled by the artist, and the broadcast engineer furnishes the technical means of transmitting the entertainment to the listener's home.

If the broadcasting equipment were capable of reproducing faithfully all of the frequencies which are audible to the human ear, and also the extremes of volume or loudness range creatable by a large orchestra, there would be overlapping of the functions to be performed by the artists and by the engineers only in regards to acoustics and microphone placing. Such an ideal situation, however, does not exist, since the frequency range is somewhat limited and the volume or loudness range quite considerably limited.

The limitation of frequency range certainly affects the listener, but does not markedly affect the cooperation between the artist and the engineer. The limitations of volume range, however, not only affect the listener but markedly affect the cooperation between artist and engineer, since the manner in which the volume range is compressed may very greatly change the aesthetic content of the musical entertainment.

This paper describes the acoustic and volume control conditions surrounding the broadcasts of the Philadelphia Orchestra under the direction of Leopold Stokowski, during the winter season of 1931 and 1932.

The aim of these broadcasts was acoustically to transport the listener from his home to the orchestra hall so that he might enjoy the concert as would a member of the audience. This aim is not to be considered ultimate, for it is possible that as the art progresses, new forms of entertainment, different from any which can now be witnessed in auditoriums. may be developed for electrical transmission and recording.

The paper deals mainly with the two following subjects: First, acoustic conditions and method of placing the microphone; second, method of "mixing" or volume control followed to insure the maximum artistic effect possible with the reduced volume range.

ACOUSTIC CONDITIONS AND MICROPHONE PLACEMENT

There is little possibility of artificially controlling the acoustic conditions of the auditorium for a broadcast of this type, since the music

is produced and picked up in the presence of an audience. Such acoustic control as can be accomplished is largely in the nature of microphone placement in order to best use the existing situation.

Preliminary to determining the microphone position, measurements were made of the reverberation time in the Academy of Music in Philadelphia, first with the hall empty but the orchestra shell set up on the stage, and second with the audience in their seats and the orchestra on the stage. These latter measurements were made through the cooperation of Mr. Stokowski and the audience at the beginning of the intermission of an actual concert. Fig. 1 shows the values of reverbera-

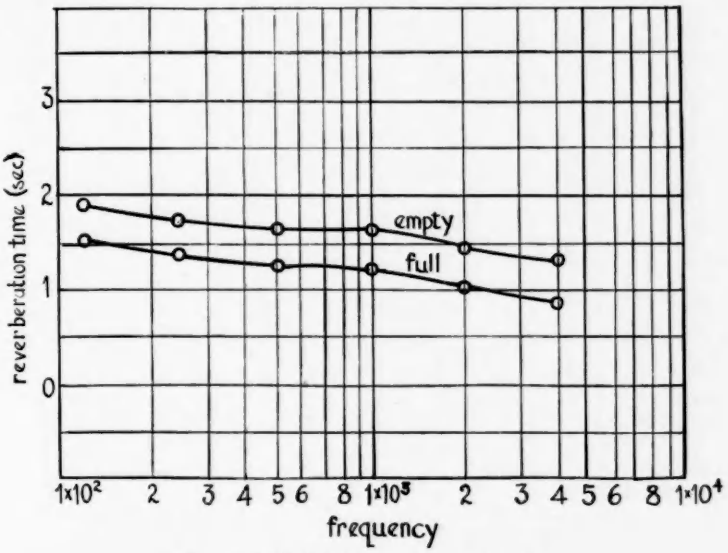

FIG. 1. *Philadelphia Academy of Music.*

tion time obtained under these two conditions, plotted as functions of frequency.

The reverberation time with the audience present fortunately fell within the band of best auditorium conditions for recording and broadcast pick-up.[1] This fact insured the possibility of placing the microphone far enough away from the orchestra to get a musical blend similar to that heard by a member of the audience, without causing the reproduction to sound excessively reverberant.

The next problem was to determine the best microphone position from the point of view of the artistic effect of the music. Theory, and

[1] J. P. Maxfield, *Acoustic Control of Recording for Talking Motion Pictures*, Journal Society of Motion Picture Engineers, January, 1930.

previous recording experience in the empty auditorium, indicated the probably best position at about the distance of point *C*, Fig. 2. It was impossible to locate the microphone further from the orchestra than point *C*, without approaching too closely to the audience, or without placing it in the area of sound interference reflected from the dome in the ceiling. To check this position three microphones were hung at

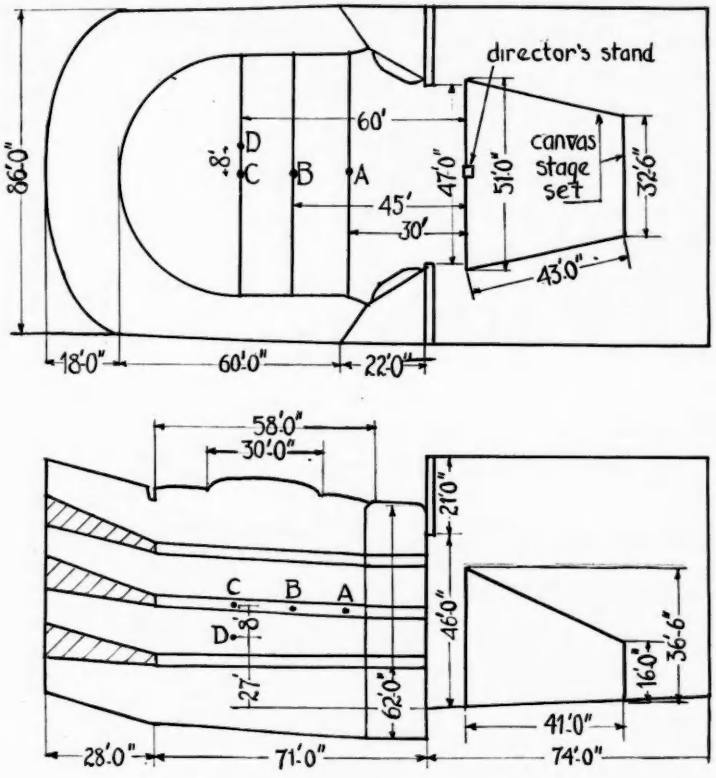

FIG. 2. *Philadelphia Academy of Music.*

positions *A*, *B* and *C*, respectively, and transmission of music from these three positions was compared, during a concert, by Mr. Stokowski. Position *C* was picked as being best.

In general, rather severe interference phenomena were present all along the center line of the hall, and after determining the best average distance, it was necessary to explore rather carefully positions around this distance to find that position in which the pick-up was most "free" and natural. One such position was found to be located at point *D*, Fig. 2. A single microphone in this position operated satisfactorily, as

anticipated, so for most of these broadcasts there was no "mixing" of several microphones during the musical part of the program. There was, however, one concert in which solo artists were used and in this case an additional microphone was used for the singers. This microphone was placed about twelve to fifteen feet in front of the singer, who in turn was another twelve to fifteen feet in front of the orchestra. During a solo part, this microphone was "faded" in until the accentuation of the voice, brought about mainly by the increased apparent closeness, was such that it gave the illusion that a listener in the hall would expect to receive, aided by his ability to listen binaurally and also to pay particular attention to the singer by visual means. It has been our general experience that this ability to see the artist is of very real help, at a concert, in enabling a listener to apparently hear the voice stand out above the orchestra.

In all broadcasting this effect must be compensated for, otherwise the voice of the singer appears to become submerged in the accompaniment. Had this type of broadcast been handled from the studio where the arrangement of the artists is under the control of the broadcaster, the increase in nearness required for proper balance should properly have been obtained by having the singer somewhat nearer to a single microphone than was the orchestra. Two microphones are used simultaneously in this manner only under conditions where control over the arrangement of the musicians, or some other difficulty, require the use of a multiple pick-up system.

There are three important results which arise from the use of a microphone at a distance from the orchestra sufficient to blend the reverberant sound with the direct sound. The first result is the roundness and fullness of the tone with which the individual or ensemble instruments are reproduced during sustained notes.[2]

The second effect has to do with adding to the music sufficient reverberation to cause the listener to appreciate the size and acoustic character of the auditorium in which the concert is being played. It is believed that this recognition by the listener is largely responsible for the feeling that many listeners have that they have been acoustically transported to the auditorium.

The third, and the most important from the viewpoint of operating technique, is the increased apparent volume range permissible. With the microphones very close, the larger part of the sound energy falling

[2] R. L. Hanson, *Liveness of Rooms*. Read before a Meeting of Acoustical Society of America, November 30, 1931.

upon them is the direct sound from the instruments to the microphones, whereas in the more distant position a considerable portion of the energy is reverberant. For the position *D*, Fig. 2, it has been computed that the ratio of the average energy intensity of the reverberant sound to the energy intensity of the direct sound is of the order of 5 to 1 or more. Under such conditions it is readily seen that the volume or energy variation brought about by the individual blows of the heavy percussion instruments, and even sudden loud passages by the orchestra, as a whole, produce a smaller change in total intensity than would be the case were most of the sound direct. This distant pick-up does not, of course, change the intensity range existing between soft sustained notes and loud sustained notes.

It is thus possible, without working outside the limitations of the equipment, to broadcast the music more nearly as it is played than would be the case if the microphone were situated very close to the orchestra. In this connection it should be noted that this is not a means of compressing the volume range, since the best seats in the auditorium are at least as far away as the microphone, and the audience is therefore receiving this effect itself.

VOLUME CONTROL

In spite of the assistance rendered by this distant pick-up, it is still necessary to change the range between the very soft passages and the loud ones, in order that the music shall not sink into noise on the soft passages or overload the receiving sets on the loud passages. As has been pointed out in the literature,[3] there are two ways open to the engineer by which this compression of the range may be accomplished. The first of these, and the more common one, is, in the case of a crescendo, to permit the volume to rise fairly close to the overload level, and then begin compressing, i.e., lowering the volume by means of the volume control dial. This is the natural method available to the mixer who is not familiar with the music, but it unfortunately removes a great deal of the emotional value from the big crescendos. In a similar manner, failure to raise the volume level until a diminuendo is becoming so low in level that it approaches noise is also harmful to the emotional effect.

The second method insures retaining a maximum of the artistic effect possible, but requires knowledge on the part of the mixer of the music that is being played. This technique is as follows: As a crescendo

[3] J. P. Maxfield, *Recording for Sound Motion Pictures*, page 267, McGraw-Hill, 1931.

or diminuendo is approached, the mixer gradually changes the volume transmitted until he has reached a value which will be safe for the remainder of the passage. The mixer dial is then left unchanged and the final part of the crescendo or diminuendo is transmitted undistorted.

This second method, as described, still leaves the judgment of when and of how rapidly to operate the dial in the hands of the mixer. In order to put the volume compression under the control of the conductor, the technique employed in connection with these broadcasts was developed. During the Friday afternoon concerts, the control dial was operated in accordance with method one throughout the concert. Notes were made on a musical score, indicating the measure at which a change had to be completed in order to avoid either overload on the loud passages or submersion in noise on the softer passages. There was also noted on the score in decibels, the necessary change of control dial setting. This marked score was then submitted to Mr. Stokowski, the conductor, who indicated on it the place where the change should be made and the rate at which it should be made. During the broadcasts, which occurred Saturday evening, the dial settings were changed in accordance with this marked score, the operator having nothing to do but read the score and keep his settings corresponding with the indications.

It was the consensus of opinion of numerous observers who compared these broadcasts with others, in which it was known that such precautions were not taken, that the apparent volume range was very much greater and the emotional content of the music greatly enhanced.

The amount by which the volume had to be compressed was unexpectedly small. In the vast majority of selections, a change of 10 to 15 db was the maximum change required, and only one concert required as much as 20 throughout its whole playing. When it is remembered that the range of a symphony orchestra is of the order of 55 to 60 decibels as measured by the volume indicator, it is seen that this compression is not exceedingly large.

While volume compression handled in this manner leaves a maximum of artistic effect in the music, it does fail to transmit the full emotional value which would have been received had the full range been used. It is therefore highly desirable that every effort be made in the future to so improve the broadcasting sets and facilities that a full volume range can be transmitted undistorted. In the matter of possible added improvement artistically, the necessary increase in frequency range should not be overlooked, since from the listeners' point of view this

probably contributes as much to the aesthetic value of the music as does the increased volume range.

The same principles of control are applicable to transmission of programs from a broadcasting studio, with the necessary changes in quantitative factors to compensate for the difference in the size of the studio and nature of the program. In the case of the studio, the acoustic control is completely in the hands of the broadcaster; first by proper acoustic design of the studio[4] and secondly by the proper arrangement of the musicians and the microphone.

The studies and experimental work connected with the Philadelphia broadcasts were carried out with the close cooperation of Mr. Leopold Stokowski, conductor of the Philadelphia Orchestra, engineers of the Columbia Broadcasting Company, and engineers of the Bell Telephone Laboratories, Inc.

August 18, 1932

[4] *Acoustics of Broadcasting and Recording Studios*, G. T. Stanton and F. C. Schmid.

ON THE ACCURACY OF THE AURAL METHOD OF MEASURING NOISES[1]

By Juichi Obata and Sakae Morita
Physics Division, Aeronautical Research Institute, Tôkyô Imperial University

Introduction

The problem of noise has of late been the subject of much attention, and a great deal of work has already been published in regard to it in this Journal and in many other publications.

So far as the authors are aware, very little has, however, been discussed regarding the accuracy of the measurements. Especially, the accuracy or reliability of the aural method of measurement is usually overlooked. Although the method is widely employed in various fields of noise measurements, the idea is prevalent that the method depending upon our auditory sensation is unreliable and of little scientific value.

According to our own experience, however, the aural method of measuring noise, unreliable though it may at first appear, will give fairly satisfactory results, provided it is carried out with necessary care. It may therefore not be out of place to discuss here in detail the accuracy of the aural method by means of data which we recently obtained in actual noise measurements.

1. Experiment

The authors recently have had opportunities to measure the noises in one of the suburban trolley cars of Tôkyô and also in the Tôkyô Subway. The principal object of these experiments was to study the effects of a particular kind of car and of rail bed on the character of the noise, and they were carried out with the cooperation of the Institute of Civil Engineering and the Aeronautical Research Institute, both of the Tôkyô Imperial University, the Electrotechnical Laboratory of the Department of Communications, and the Electrical Research Institute of Tôkyô Municipality. Vibrations of the car were also measured by experts of the Earthquake Research Institute of the Tôkyô Imperial University. The following instruments were used for the noise measurements:

[1] The expenses incurred in the prosecution of these studies was met with the subsidy granted the senior author (J.O.) by the Hattori Hôkôkwai.

	Suburban trolley car	*Subway*
Geräuschmessers[3] (Siemens and Halske)	3	2
2-A audiometers (Western type)	2	2
Indicating noise meters, consisting of a microphone, an amplifier, and a milli- or microammeter	1	4

Throughout the present experiments, the noise measurements were carried out simultaneously with all the instruments working at intervals of ten seconds, by means of electric lamp signals.

In discussing the accuracy of the noise measurements by the aural method, we shall confine ourselves at first, for the sake of brevity, to data obtained by the Geräuschmessers. The suburban trolley car runs over a ten-mile tract west of Tôkyô. Its rails, over which the measurements were made, lie almost in a straight and horizontal line through open fields and with neither buildings nor disturbing objects in the neighborhood. Single as well as coupled cars were tried, the cars making several to-and-fro trips between two stations through five intervening stations. About 125 measurements were made with each instrument.

To measure the subway noise, a car with the instruments set up in it, was linked to a passenger car and several trips back and forth made between Asakusa and Kanda, the two terminal stations at the date of the measurements, through the stations Uyeno, etc. Noise measurements were made inside the car as well as on the station platform. About 900 measurements were made in the car with the windows closed and 100 with the windows open. The platform measurements total about 100.

2. Factors Affecting the Accuracy of the Measurements

Although several factors affecting the accuracy may be quoted, they may be after all reduced to the following items:—

(a) Instrumental errors

Every instrument has its own characteristic, more or less pronounced: The pitch as well as the wave form of the standard sound differs considerably with each instrument, and moreover, the intensity of the standard sound corresponding to the same indication may not be exactly the same by different instruments.

[3] All the Geräuschmessers employed in the present experiments are of the old type, having the so-called "alte phon" or "phon" scale, as defined by H. Barkhausen.

(b) Personal errors

Personal errors may be divided into two classes. In one we include what we shall call permanent errors, that is, errors inherent in a particular individual person, such as abnormal hearing, unequal audibility of the two ears, and other physical disabilities. In the other class we include what we shall call temporary errors, that is, errors due to temporary mental and physical states of the observer, and also to his biased anticipations of results.

(c) Errors arising from marked differences in the nature of the noises and in the conditions and methods of measurements

In a broad sense, these errors may be classed with personal errors mentioned above.

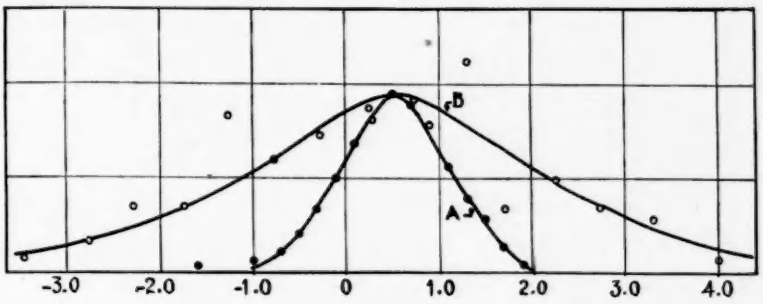

FIG. 1.

3. CALCULATIONS AND DISCUSSIONS

(a) Error curves

Since, as already stated, hundreds of data have been obtained for the subway noise, error curves have been drawn with these data as bases. The object was to know the forms of the error curves themselves, and also to deduce graphically a relation between the various conditions surrounding the experiments and the accuracy of the observations.

In Fig. 1, the abscissa is the difference between the data obtained by two Geräuschmessers, and the ordinate the number of observations corresponding to that difference. Curve *A* corresponds to measurements made in the car with windows closed and curve *B* to those made on the platforms of the station.

It will readily be seen that the points lie fairly well on the error curve in the usual sense, especially in curve *A*, owing undoubtedly to the large number of observations that were made. In this connection, it is

very interesting to find that even the data of the aural method of measuring noise, a method often considered unreliable, are governed by the law of errors. Another point to be noted is that, curve A, as might be expected, is much sharper than curve B. Obviously, the noise-level of a sound which is comparatively steady, or which is changing smoothly, as in the case in the car, can be determined much more easily than in the case of sounds, such as on the platform, which are changing rapidly through a wide range.

(b) Errors due to the nature of the sound

The most frequently occurring difference between the values obtained by the two observers, namely, the value corresponding to the vertex of the error curves in Fig. 1, amounted to about 0.5 phon. It may be attributed partly to instrumental and partly to personal error. If it was mainly instrumental, the value should be constant, whatever may be the nature of the sound. If, on the other hand, the error was mainly personal, then the value should differ with different sounds. Although the abscissae of the vertices, or the differences corresponding to the vertices, of curves A and B seem to coincide well with each other, (the ordinates being drawn to the same scale), it would be unsafe, in view of the great flatness of curve B, to conclude immediately that the difference is independent of the nature of the sound. Much closer study of the data is necessary before the exact cause of the difference can be known.

In the Tôkyô Subway, the rail bed is not the same throughout the line. It is ballast between Asakusa (one terminus) and Uyeno, and concrete between Uyeno and Kanda (the other terminus at the time of the experiment). Hence, in either side of the Uyeno station, the sounds differ in pitch as well as in loudness: On the ballast bed the noise is weak and low-pitched, while on the concrete bed it is strong and high-pitched.[3]

Besides this difference in the rail beds there are other differences, such as in the speeds of the cars, the slopes or curvature of the rails according to the particular section of the tract. Thus, the noise-level is not the same for all sections.[4] In order to know such differences, the mean values of the differences of the data obtained by the two observers were calculated for each section and plotted as dots and circles in Fig. 2.

Further, the mean of the mean values has been computed for each

[3] The differences in the character of the noises on these two kinds of beds were also studied but its detailed description, being beyond the scope of the present paper, is not given here.

[4] By the word "section" is meant the part between two adjacent stations.

section and is shown in Fig. 2 by the full and the dotted lines. The former shows the run from Kanda to Asakusa, and the latter in the opposite direction. It will be seen from these results that marked differences in noise-level are found in different sections. The noisiest section is that part between Kanda (K) and Suyehirocho (S), which is undoubtedly due to the fact that the character of the noise is modified

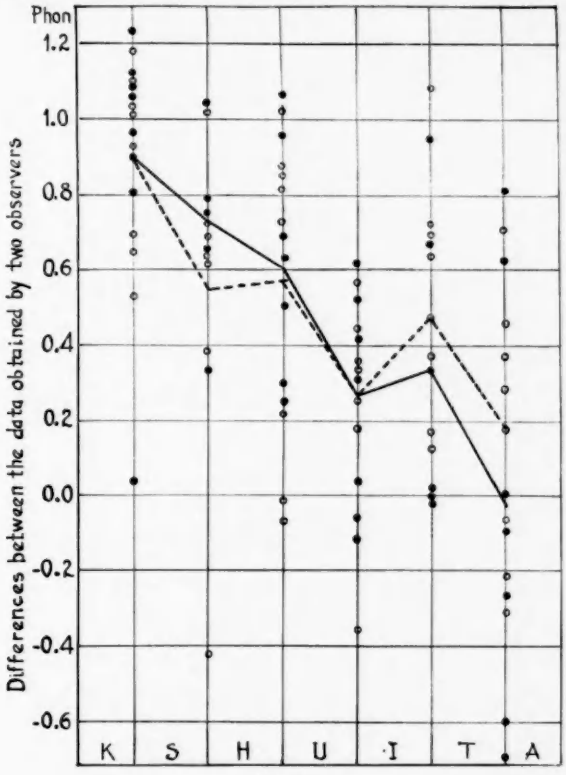

FIG. 2.

considerably in this section by the slopes that occur before entering and after leaving the tunnel which goes under river bed. In this extreme case the difference between the data obtained by the two observers was considerable, about 1.0 phon. It may therefore be concluded that the difference corresponding to the vertex of the error curves is not the instrumental error, but is chiefly due to personal errors caused by the difference in the nature of the noise. Thus, strictly speaking, the curve *A* in Fig. 1 is not a simple error curve, but in reality an envelope, or the resultant, of curves having their vertices situated continuously in different places.

(c) Effects of fatigue and proficiency on the part of the observers

As the noise measurements in the subway were carried out by the same observers for several hours, continuing from morning till evening, fatigue effect was bound to appear in the measurements in some form or other. Hence, to show this effect, the parallelism of the data obtained by the two observers was taken as the measure, with the expectation that it would be lost as the fatigue increased.

Since individual data are liable to be vitiated by various temporary disturbances, only those observations in which the value of the changes in noise-level from one observation to the next (10 seconds afterward)

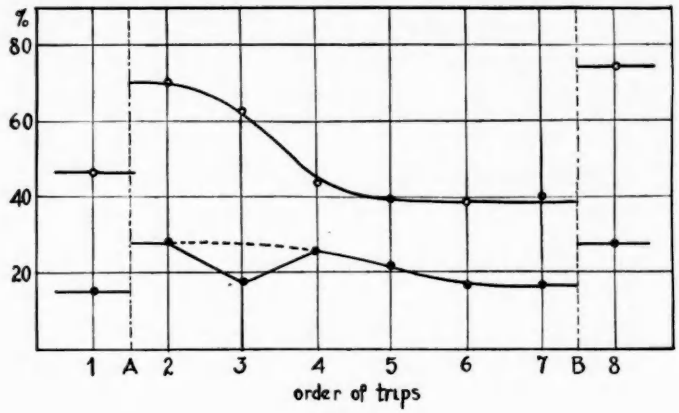

FIG. 3.

obtained by the two observers agreed to within 0.5 phon were accepted as being correct. Observations that exceeded this limit were considered incorrect, and the ratio of the number of such incorrect observations to the number of total observations were computed for each trip back and forth. The results are shown by the dots (the lower curve) in Fig. 3.

A few trial runs were made before 1, and observations interrupted for a while at *A* for luncheon and recess, while at *B* the windows of the car were opened. The upper curve connecting the circles has the same meaning as the lower curve connecting the dots, except that it is related to larger changes. In this case the mean changes exceeding 0.5 phon were selected out, and the parallelism of the data obtained by the two observers was calculated in the same way as already described.

It will readily be seen from these curves that personal proficiency, or skill, has more to do with accuracy than fatigue, at least within the limits and nature of the noises so far concerned. Hence, contrary to

expectations, the observations became more accurate the longer they were continued under identical conditions. But with changes in the conditions of the experiments, the accuracy abruptly decreased. It has thus been shown that accuracy in noise measurement is markedly increased by the observer's familiarity with the characteristics of the sounds, which is evidently one of the reasons why the accuracy differs considerably according to the nature of the sound itself.

(d) Order of the probable errors

As the next step in the study of the accuracy of the aural method, the probable errors were calculated from the data obtained with the suburban trolley car. As already mentioned, these cars when running through open fields give rise to comparatively little noise, the noise-level being about 60 to 65 decibels under ordinary running conditions. Now with the data obtained with the three Geräuschmessers, the probable error of a single observation was computed and found to be about 0.35 phon, without taking into account either instrumental error or the personal error inherent in each observer.

Table I shows the mean noise-level in each car as measured by three Geräuschmessers G_1, G_2, and G_3. d is the deviation of the value obtained by each instrument from the mean, and d_m the mean of d's.

TABLE I.

No. of observation		1	2	3	4	5	
No. of car		101 Single	105 Single	107 Single	107 Single (repeated)	105 & 107 Compound	
Mean noise-level (phon)	G_1	7.92	8.75	8.51	8.51	8.60	
	G_2	7.19	7.26	7.30	7.21	7.53	
	G_3	7.33	7.50	7.65	7.65	7.68	
Mean of G_1, G_2, G_3		7.48	7.84	7.82	7.79	7.93	d_m (Mean of d)
d (Deviation from mean)	G_1	0.44	0.91	0.69	0.72	0.66	0.68
	G_2	−0.29	−0.58	−0.52	−0.58	−0.41	−0.48
	G_3	−0.15	−0.34	−0.17	−0.14	−0.26	−0.21

Further, taking d_m as the systematic error inherent in the instrument and in the observer on that day, and after subtracting d_m from each observed value, the probable error of the single observation was com-

puted and found to be less than 0.2 phon, a rather surprisingly small value.

The probable error estimated by the error curve *A* in Fig. 1, which was obtained from data from the subway, seems to be about the same order. As might be anticipated from the flatness of curve *B* in Fig. 1, the magnitude of the probable error greatly depends upon the nature of the sound.

(e) Accuracy in recognizing variations in the sound intensity

Finally, the accuracy in recognizing variations in the sound intensity was examined with the data obtained on the suburban trolley car. The problem is to ascertain the extent to which a person can correctly recognize variations in the sound-level.

In the suburban trolley car, objective measurements of the noise-level were also made by means of a simple noise-meter, consisting of a microphone, an amplifier with a valve-rectifier, and a milliammeter. In our experience, the sound-level of a trolley car running on straight and horizontal rails in a open field fluctuates mainly with the speed of the car ($\log I \propto$ car speed, I being the sound intensity), while the nature of the sound is only slightly affected by changes in car speed. In such conditions, the simple indicating noise-meter, just mentioned, might be of value in that it indicates variations in the sound-level fairly correctly. If it does not indicate the accurate magnitude itself, it will at least indicate the direction of the change, i.e., the sense in which it may be increasing or decreasing. Hence, assuming that the meter indicated the correct direction of the change, the results of the aural measurements are investigated in comparison with it. The results are given in Table II, which shows the ratio of the number of observations indicating erroneous sense to the number of total observations.

Of the aural measurements made by the three Geräuschmessers and the two 2-A audiometers (total number of data about 750), about 35 percent was found to be in error in indicating the sense of the change. This large percentage of error in indicating the sense of the change may, at first sight, seem almost fatal to the aural method, yet it may be said to a natural result if the magnitude of the change is taken into account. Namely, of the errors of 35 percent, only 5 percent represents large changes, or those at which the mean change of the three Geräuschmessers exceeded 0.4 phon. Thus, if only changes exceeding 0.5 phon are concerned, the observations may be said to be in fairly good agreement.

The correctness of the indications of the noise-meter, mentioned

TABLE II.

Instrument / No. of obs.	G_1	G_2	G_3	A_1	A_2
1	0.26	0.22	0.20	0.48	0.24
2	0.37	0.43	0.33	0.41	0.33
3	0.25	0.25	0.39	0.32	0.34
4	0.43	0.27	0.36	0.50	0.48
5	0.48	0.25	0.40	0.37	0.33
Mean	0.351	0.284	0.336	0.416	0.344
		Total mean = 0.346			

G_1, G_2, G_3 = Geräuschmessers A_1, A_2 = 2-A audiometers

above, was also examined, using the same data. For this purpose, the sense indicated by the instruments for aural observation was taken as reference, when indications of four of the instruments out of the five showed the same signs. Thus, about 10 percent of the indications of the noise-meter was found to be in error.

4. SUMMARY AND CONCLUSIONS

Utilizing a considerable number of data that were obtained in actual noise measurements in electric cars, surface as well as underground, the accuracy of the aural method of noise measurement was scrutinized from various points of view. A rather unexpected high accuracy was found in the aural method, and it is concluded that noise measurements by the aural method, if conducted with the necessary care, have a sufficient meaning and an important value.

Finally, it may be worth noting that the indicating noise-meter, especially the one in which the frequency characteristic was made to correspond to the hearing-curve, as is so often done by means of a weighting net work, is not, contrary to its apparent trustworthiness, free from error in measuring a composite sound in which the masking effects of the various sound components play an important role.

Tôkyô, July, 1932.

POSITION FINDING BY UNDER-WATER SOUND SIGNALS

By B. R. HUBBARD
Director of Laboratory, Submarine Signal Company

More than 100 years ago, in the year 1827 to be exact, two scientists, Colladon and Sturm, carried out an experiment to determine the velocity of sound in water at Lake Geneva. From one of two boats moored a known distance apart, a bell was struck in water and simultaneously a charge of gunpowder was exploded in air. At the farther boat, an observer noted the time interval elapsing between the appearance of the flash and the sound of the bell as transmitted through the water.

A very little consideration of what was involved in the experiment of Colladon and Sturm reveals its applicability to the important problem of finding the position of a ship at sea in fog or thick weather, and in darkness. Had these two scientists started with a knowledge of the velocity of sound in water, they could have determined the distance separating the boats from the time interval required for the sound to traverse the distance.

It seems hardly possible that either Colladon or Sturm realized that the principles which they utilized to determine the velocity of sound in water would later become the scientific basis for a number of methods for finding the position of a ship at sea. However that may be, there are in operation today several systems for finding the position of a ship at sea by means of under-water sound signals. There are, for example, synchronous signals, radio acoustic sound ranging, echo depth finding, under-water sound spotting and under-water sound direction finding, all of which are being used to locate the position of a ship at sea. The difference between the various systems of position finding is largely a matter of application and not of the principles involved. Commercial ships are using synchronous signaling to fix their position. The hydrographic survey services of the world use either radio acoustic sound ranging or under-water sound spotting to locate the position of their vessels during off-shore sounding operations. The under-water sound spotting system is used also to determine the point of impact of projectiles fired from sea coast artillery. Vessels of all classes are using echo depth soundings for checking the ship's position. Finally, under-water sound direction finding has proved very useful for military purposes in the detection and location of invisible ships. Each of these systems is

fulfilling its purpose with an accuracy which usually is substantially as good as the visual system which it replaces and has an advantage over the corresponding visual system in that it may be used in times of poor visibility when the visual system is useless.

It is the purpose of this paper to describe some of the more important technical aspects of these methods of position finding by under-water sound signals.

POSITION FINDING BY SYNCHRONOUS SIGNALS

The term "synchronous signaling" is used to designate that system of signaling which is based upon the principle that the distance of a point from some reference position can be determined by measuring, at this point, the difference in the times of arrival of signals from the reference position that have been created simultaneously in two media having different velocities of propagation. Such signals, traveling at different rates, enable the distance to their source to be determined by a measurement of the gain in time of the faster upon the slower traveling signal. This method of signaling is now used extensively in times of obscured vision for finding the distance and position of a ship at sea with respect to a lightvessel or a danger point.

This method of measuring distances involves, necessarily, the emission of signals propagated with different velocities. Several kinds of signal are available for this purpose at sea. We have, for example, light and radio signals, both of which may be regarded as propagated practically instantaneously; air borne sound signals which travel about 330 meters a second; and under-water sound signals which travel about 1500 meters a second. Thus, three combinations are available—air and under-water sound signals, radio or light and air sound signals, and radio or light and under-water sound signals. The aerial sound signals may be those produced by a fog horn while the under-water sound signals can be produced conveniently by the submarine bell or the submarine oscillator.

Of these three combinations, all are not, however, equally reliable in practice. The use of the light-flash has, of course, the drawback of being invisible when synchronous signals are most useful to the mariner, that is, in thick or foggy weather. Hence, only under certain conditions is the combination of synchronized light and sound signals of value.

There are limitations, also, to the use of sound signals propagated through the atmosphere. These limitations arise from the fact that the audibility of such fog signals in air varies in a most capricious manner.

A detailed discussion of the influence of atmospheric conditions upon the audibility of fog signals may be found in several papers* on this subject; only a few of the more important limitations of the sound-in-air signals for synchronous signaling purposes need be mentioned here. The uncertainty and unreliability of the ranges of even the most powerful fog horns that have been used up to the present time are well known. These signals are apt to be difficult to pick up in strong weather, especially when the wind is blowing from the ship towards the source of sound. The sound appears to be weakened by these conditions, and if the wind is making much noise on the ship, it may not be heard at a sufficient distance. Furthermore, the sound signals must be listened for in the open. A more serious limitation, however, is the strange though, fortunately, not too common phenomenon of silent zones. A ship may be, for example, well within the range of audition, the weather may be calm, even windless, and the sound may be mute over certain areas. In these instances, there appears to be such a deviation of the sound as causes it to rise and arch over certain areas. In such cases, the value of the synchronization is lessened not only because of its unreliability but also because the sound which is heard beyond the silent zone will not have traveled directly from the source. Probably the increase in distance is not much; nevertheless there may be an appreciable error here.

Water, on the other hand, has many important advantages over air as a means for transmitting the sound signals. For one thing, water is free from the dangerous zones of silence. In addition, there is the well-known fact that sounds travel under water with remarkably little loss of clearness or intensity. The absorption of the sound is much less in water and consequently the sound signals can be transmitted to a distance many times greater than when they are transmitted through the air. Under favorable atmospheric conditions the average range of the submarine sound signals is from two to three times that for the aerial sound fog signals; while under unfavorable conditions the range of the submarine oscillator signals is ten or more times that for the air borne fog signals.

It is evident, therefore, that, of the three modes of synchronous signaling which were suggested, the combination of under-water sound

* "The Fog Horn as an Aid to Navigation." B. R. Hubbard, Transactions of National Safety Council, 1931.

"Influence of Atmospheric Conditions upon the Audibility of Fog Signals." B. R. Hubbard, Jour. Acous. Soc. 3, 111 (1931).

and radio signals remains as the one most free from liability to failure. Furthermore, this method of synchronous signaling is a logical development in extending the usefulness of the radio and submarine signal systems, already installed, to the mariner. Independently of each other, submarine signals and radio are now used extensively to give the mariner the direction of the source from which the signals emanate. Led by the fact that these two systems can be combined by means of a simple synchronizing device to give distance finding signals, many of the lighthouse authorities of the world have installed synchronous signaling systems at a number of the most important navigational points.

This method of distance finding is not a recent development, for, as far back as 1923 the Nantucket Lightvessel was equipped by the U.S. Lighthouse Board with synchronized radio and submarine signals. Later as the lightvessels along the coast of Europe were equipped with submarine oscillators, these installations were combined with radio in order to give distance by synchronous signals. At the present time there are 28 stations which, by the combined use of radio and submarine signals, are equipped to give mariners the benefit of distance finding.

To make available this aid to navigation, it is merely necessary that lightvessels should be equipped for sending synchronized radio and submarine oscillator signals. To make distance finding observations possible on ships equipped with submarine signal receiving apparatus, no new apparatus is required other than a circuit arrangement whereby the same observer may hear both the radio and the synchronized submarine signals and a watch wherewith to measure the time interval between the arrival of the two signals at the receiving ship. A watch, a stopwatch, or a special form of a stop-clock with a dial graduated in miles will serve to indicate the distance between the observer and the source of the synchronized signals. Fig. 1 shows this method of distance finding in operation on a lightvessel.

The most commonplace, and often one of the most urgent of the problems which confront the mariner is the determination of his position upon approach to the coast. Synchronous signaling is the modern method of supplying this information. From the data as to his distance combined with a radio or submarine signal bearing, or from the data furnished by two distance determinations separated by a known time interval combined with the course and distance run by the vessel during this interval, it is a simple matter for the mariner to determine his position with ease and accuracy.

POSITION FINDING BY RADIO ACOUSTIC SOUND RANGING

The use of under-water sound and radio signals for distance finding is not limited in its application, however, to the problem of finding the position of a merchant ship at sea. A somewhat similar system of position finding, known under the names of "subaqueous sound ranging" and "radio acoustic range finding" is used by the U.S. Coast and Geodetic Survey for fixing the position of a survey ship with reference to a sounding which is being taken.

The geographic position of a sounding is just as important as the depth, for measurements of ocean depths are of no value in a survey unless the geographical positions of the soundings are accurately de-

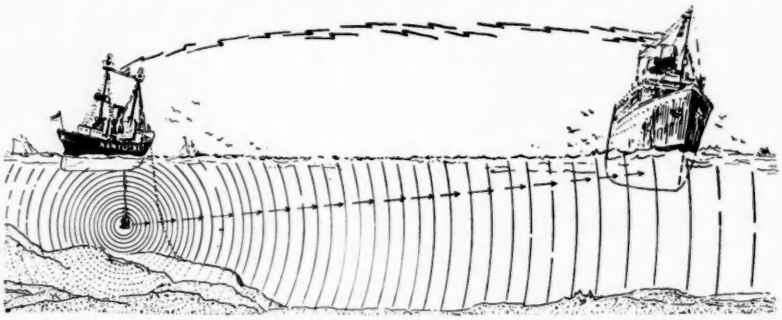

FIG. 1. *Synchronous signaling.*

termined and unless the soundings are properly related to one another. In fact, the problem of locating accurately the soundings in an offshore region that may be pervaded with fogs and strong currents is by far one of the most difficult problems connected with this class of work. In coastal surveys, positions may be fixed by sighting on shore signals, or on buoys whose positions have been determined. These visual fixes, of course, require clear weather as do also astronomical observations. On some coasts of the United States fogs and haze prevail over long periods, punctuated by only short intervals of clear weather. In such localities, dependence upon visual fixes obviously would hamper the progress of hydrographic surveys to a considerable degree.

The recently developed method of radio acoustic sound ranging permits the prosecution of hydrographic surveys during foggy or rainy weather and during the hours of darkness. As compared with the methods formerly used, the U.S. Coast and Geodetic Survey considers subaqueous sound ranging to be a "more accurate and far more ex-

peditious method of fixing the position of soundings beyond the range of visibility of shore objects." Due to the fact that this method can be used during hazy weather and at night, a surveying party can carry on operations continuously whenever the sea is not too rough for accurate sounding operations.

For survey work the position is fixed by measuring the time required for the sound to travel through the water from the survey ship to two or more places near shore, the exact geographical position of which are

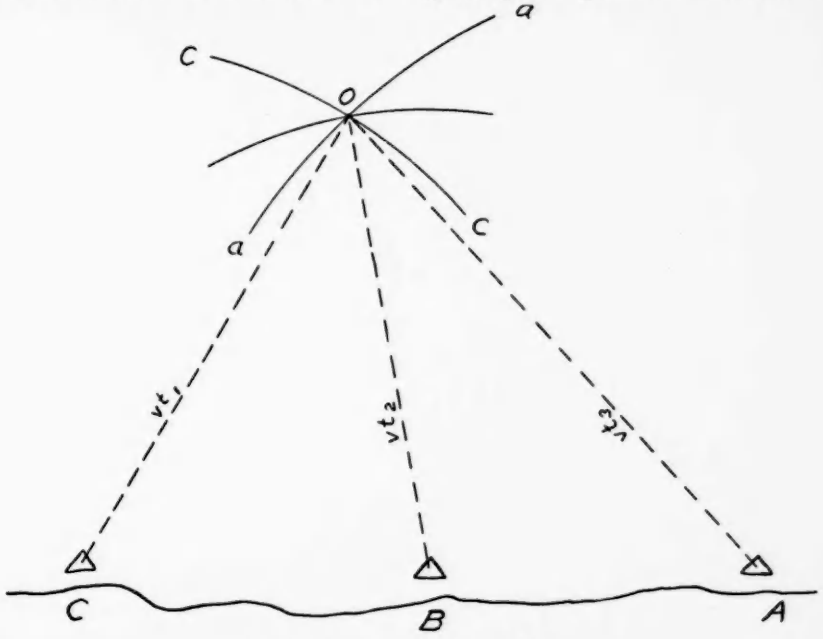

FIG. 2. *Radio acoustic position finding.*

known. These time intervals multiplied by the velocity of sound through the water give the distances from these known places, and, hence, the position of the ship.

As shown in Fig. 2 the points *A*, *B* and *C* represent shore stations which are accurately located at sites far enough apart so that the distance arcs will give good intersections throughout the area where they are required for control. The third station is provided as a check, but it is not essential. The distances *OA* and *OC* are measured by the radio acoustic sound ranging method. The intersection of the arc *aa*, of which the distance *OA* is the radius and *A* the center, and the arc *cc*, of which the distance *OC* is the radius and *C* the center, fixes the position of the ship at *O*.

In this method of position finding the following procedure is used. A small bomb of high explosive material, usually T.N.T., is detonated under water near the ship, and the instant of explosion is recorded by a time measuring apparatus or chronograph on the ship. The arrival of the sound waves from the explosion is picked up by hydrophones at the several stations near shore and reported electrically, through cables, to radio stations on shore. Each of these radio stations responds automatically to the reception of the sound waves of its hydrophone station and transmits a radio signal back to the ship's radio receiving set. The instant of reception of these radio signals at the ship is recorded on the chronograph apparatus, thus giving the time interval between the explosion and the reception of the sound at each of the hydrophone stations. The entire operation is controlled at the ship, the shore stations being automatic except for an attendant to keep the apparatus ready for operation. The time intervals can be measured to one one-hundredth of a second, and the sum of all errors is so small that the position of the survey ship can be determined usually with an error not much in excess of one percent of the distance from the shore stations.

A position 206 miles off shore has been obtained by this method, and little difficulty has been experienced in its regular use out to the limit usually fixed for hydrographic surveys, 50 to 70 miles off shore. Radio acoustic control has been used on the north Pacific coast of the United States for more than four years and, under the conditions encountered in this region, its value has been proved in every respect. It has played an important part also in the recent resurvey of Georges Bank.

The reference stations may be on shore or, as in the case of surveys far off shore such as the recent resurvey of Georges Bank, the reference stations may be anchored off shore. In the latter case, dependence is placed principally upon astronomic observations upon stars for fixing the position of the reference points. The altitudes of three or more stars, well distributed around the horizon, are measured by sextants, and a position line is computed for each star. The intersection of these lines determines the position which is either at the intersection, if all lines intersect at a point, or is at an equal distance from each line, if they form a small triangle or a several sided figure. The astronomic fixes thus obtained are used to fix the reference points to which all of the off-shore soundings are related.

This radio acoustic sound ranging method of position finding has not only increased the speed of hydrographic surveys because of its independence of weather conditions but has made possible the accurate

location of soundings far off shore where former survey methods made it difficult or impossible to chart soundings. Furthermore, it has made it possible to decrease the unit cost of surveys while at the same time giving the mariner added service.

POSITION FINDING BY ECHO DEPTH FINDING

The possibility of fixing the position of a ship by means of a series of soundings may not have occurred to those unfamiliar with the art of navigation. As a matter of fact, the mariner frequently uses this method to determine with approximate accuracy just where he is on the sea. Because of this fact great accuracy is required of the hydrographer even where the water is so deep that there is no possible danger of the largest ship afloat touching bottom.

The hydrography shown on a mariner's chart has a twofold object: First, to point out to the navigator the invisible dangers which he must avoid; and, second, to reveal the configuration of the bottom and also its physical characteristics so truly that, by the use of his sounding apparatus, he may fix his position in relation to those dangers, or when off shore or the coast line is invisible, determine his distance from land.

Position finding by echo depth finding is a comparatively simple procedure. The mariner, while proceeding on a known compass course, takes a series of soundings of the depth of water beneath his ship and thus obtains a contour of the bottom of the sea over which he is moving. Armed with this information, the mariner inspects the chart in the neighborhood of the place where he believes himself to be until he finds a line of soundings parallel with his course which corresponds with the contour he has determined from the echo depth soundings. In actual practice this is usually a comparatively simple operation and requires very little time. A single example will show how this is done. Consider the case of the navigator of a westward bound trans-Atlantic liner who has not seen the sun, moon or stars for perhaps two or three days. He is approaching a locality in the neighborhood of Georges Bank where it is imperative that he either verify his dead reckoning position or alter his course to the southward. Equipped with echo depth sounding apparatus, he notes that he has reached the Continental Shelf—the 100 fathom curve. The depths decrease for a few minutes then, suddenly they increase to 100, 200, 300 fathoms and then, just as suddenly decrease. He recognizes the feature immediately as the recently discovered submarine valley, known as Corsair Gorge which lies 161 miles, 74° true from Nantucket Light. This valley which is about two miles wide, eight miles

long, 1800 feet deep, and lies directly in the westbound, winter, trans-Atlantic track, is ideally located with relation to the track and other submarine features for fixing the position of a ship by a series of soundings taken by a ship traversing any section of it. Reassured as to his position, the navigator lays a new course to Nantucket Lightship, saving perhaps many miles of actual travel and no small amount of uncertainty and worry.

Of course the extent to which echo depth soundings can be used as an aid to determining position depends entirely upon how well the chart portrays a true contour map of the ocean bottom. Unless the submarine features are shown in their true positions and in detail, they cannot be used to full advantage for this purpose. It can be understood readily, therefore, why echo depth sounding has stimulated the demand not only for more accurate and detailed hydrographic surveys, but also for their extension seaward.

POSITION FINDING BY THE UNDER-WATER SOUND SPOTTING SYSTEM

Another problem in position finding that has been solved through the use of under-water sound is that of locating the point of impact of a projectile fired from major calibre sea coast artillery. The ability to solve this problem during hazy and foggy weather or at night is of considerable importance, for the information thus obtained enables the coast artillery man to correct the aim of his gun to bring about a direct hit.

The under-water sound spotting system used for this purpose is an adaptation of a similar system used during the World War for spotting field artillery.

The theory of this method of spotting is quite simple. When a projectile strikes the surface of the water, sound waves are created which travel in all directions below the surface with a speed equal to the velocity of sound in sea water. Three or more hydrophones are mounted on tripods and the tripods are planted at convenient but carefully selected locations along the shore. These hydrophones are connected to recording apparatus on shore by means of submarine cables. Fig. 3 shows a typical under-water sound spotting system. The relative times of arrival of the sound waves, created by the impact of the projectile on the water, at the hydrophones is recorded upon a photographic tape by suitable apparatus on shore. From this record the difference in the times of arrival of the sound waves at the hydrophones is determined.

Referring to Fig. 4 let us now assume that the time difference t between the arrival of the sound waves at hydrophones or water stations A and B has been determined as just described and the location of the point P, which is for the time being unknown, is desired. Clearly the point P may be any point whose difference in distances from the points A and B is a constant and equal to vt, where v is the velocity of sound in sea water. In other words, P must lie somewhere on a hyperbola

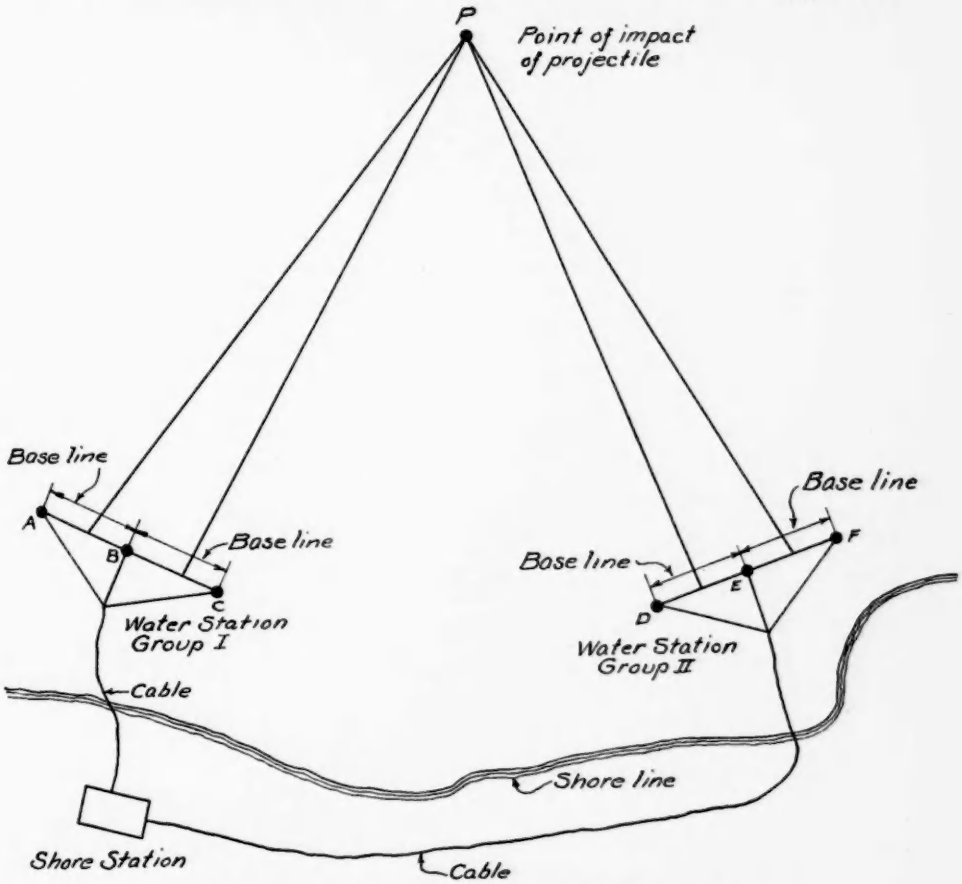

FIG. 3. *Under-water sound spotting system for locating the point of impact of projectiles.*

whose foci are A and B, for by definition, a hyperbola is the locus of a point the difference of whose distances from two fixed points, called foci, is a constant.

It will be recalled that a hyperbola consists of two symmetrical branches which approach straight lines, called asymptotes, as they go

away from the origin in each direction, and finally coincide with the asymptotes at infinity. Upon which of the two branches of the hyperbola the point P lies is easily determined, since it can always be determined at which water station the sound arrives first.

Another ambiguity would seem to exist as to whether the point P lies in front of or behind the base line joining the two points A and B. Fortunately, this ambiguity is easily removed, for the water stations

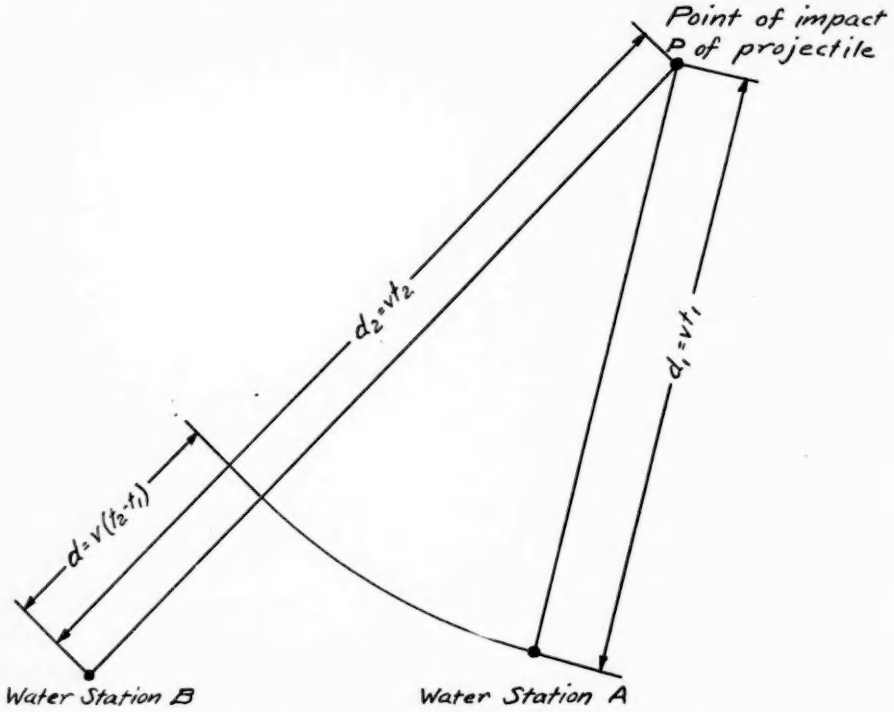

FIG. 4. $t = t_2 - t_1$. V = *velocity of sound in sea water.*

are always located so near the shore that the fall of a projectile behind them is impossible. Thus the point of impact of a projectile is always in front of the base line, and hence the area in back of the base line may be disregarded entirely, so that no real ambiguity exists.

Finally, the point of impact of the projectile is determined by measuring the relative times of arrival of the sound waves at three hydrophones and determining the point at which the two hyperbolas intersect, as shown in Fig. 5.

Unfortunately, however, the three hydrophone spotting system has a very serious weakness in practice. If, for any reason, an unsatisfactory

photographic record of the time of arrival of the sound wave at one of the three hydrophones is obtained, it is impossible to locate the point of impact of the projectile, for the data essential to the construction of one of the hyperbolas are lacking. This weakness can be eliminated

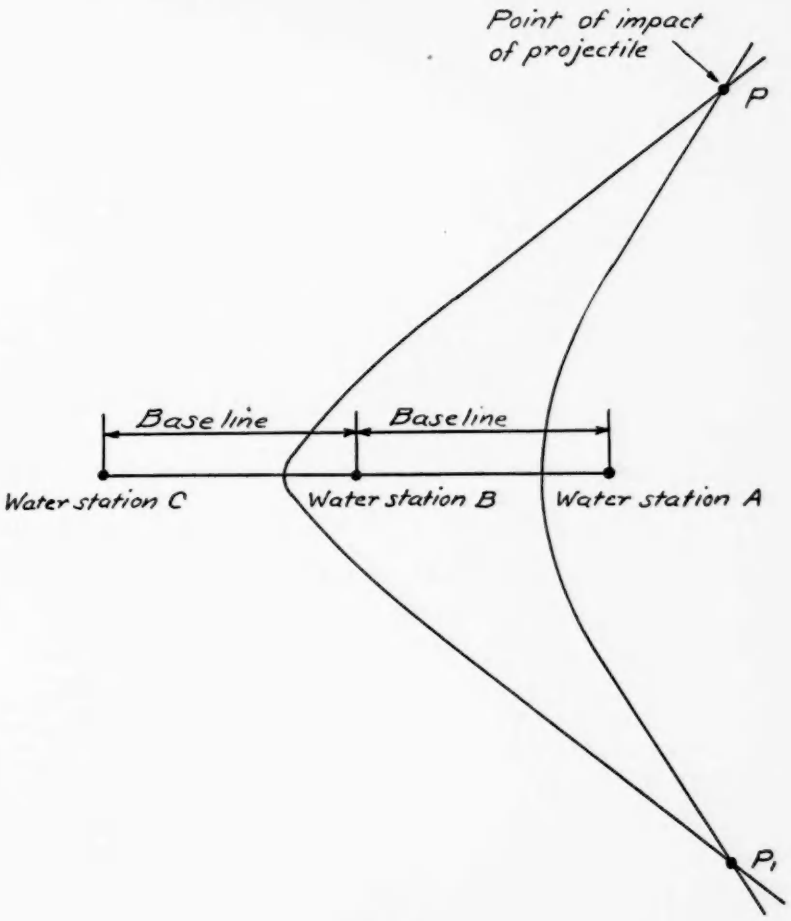

FIG. 5.

quite easily, however, by employing two groups of three hydrophones, which are distributed along the coast as shown in Fig. 3. This system insures ample verification of the point of impact, for, as shown in the figure, a fourfold intersection may be utilized under favorable conditions for locating the point of impact. If an unsatisfactory record of the time of arrival of the sound at one of the hydrophones is obtained, the

three remaining intersections may still be used to locate the point of impact.

This system, relying as it does entirely upon under-water sounds, is well adapted to locating the point of impact of projectiles from coast artillery and has the advantage over visual spotting that its operation is not hampered by haze, fog or darkness. Refinements in the technique of receiving and recording the data, combined with a personnel carefully trained in a standardized procedure, have made it possible to locate the point of impact on an artillery plotting board with ease and rapidity.

This method of position finding has been used by some of the hydrographic survey services of the world as a means for fixing the position of a survey ship off shore. In this case the under-water sound signal usually is obtained by the explosion under water of a small bomb dropped from the sounding vessel. As compared with the radio acoustic position finding method, however, this method has the disadvantages that the survey ship does not have direct control of the observations and that the results are not immediately available after the observations have been completed.

POSITION FINDING BY MULTIPLE UNIT UNDER-WATER SOUND DIRECTION FINDING

This method of position finding was developed primarily for military purposes. It differs from the methods already described in that the sound source, which in this case may be a foreign vessel, has no desire to cooperate in the problem of fixing its position. The ship must be detected first and then its position fixed solely by means of the under-water sounds which are created by the motion of the ship through the water.

For this purpose, three or more under-water sound receivers or hydrophones mounted at equal intervals in a straight line are submerged at an appropriate but known place in the sea. Referring to Fig. 6, let $H-1$, $H-2$,—$H-6$ represent a group of six such hydrophones. Suppose the plane wave front WW which originated at the distant ship passes in succession over the receivers $H-1$, $H-2$,—$H-6$ at the instant $0, t1$, $t2$,—$t5$, respectively. (It is assumed that the distance between the sound source and the hydrophones is so large in comparison with the space occupied by the hydrophone line that for all practical purposes the wave front is a plane.) If, now, the hydrophones $H-1$, $H-2$ and $H-3$ are connected together and to a telephone receiver applied to, say the left

ear, while the hydrophones $H-4$, $H-5$ and $H-6$ are similarly connected to a telephone receiver applied to the right ear, then there will be a phase difference in the resultant sounds as heard in the right and left ears and, due to the binaural effect, the sound source will appear to be to the right of the listener. By arranging the hydrophone group to rotate in an azimuth, this phase difference can be reduced to zero at

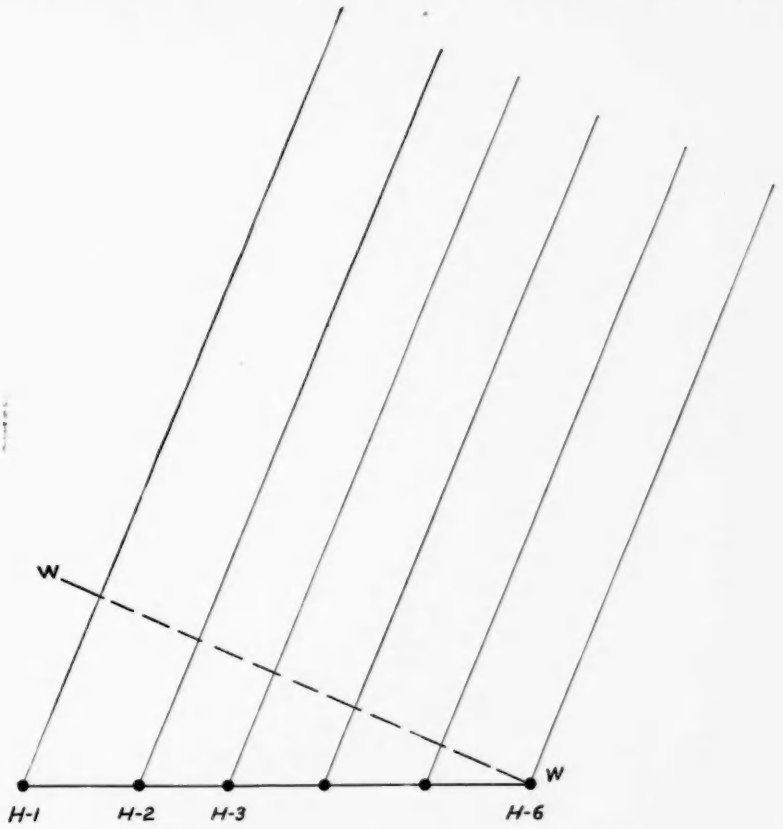

FIG. 6.

which time a binaural balance is obtained and the sound source appears to the listener to be straight ahead. Under these circumstances the sound source will lie along the perpendicular bisector to the base line of the hydrophone group. Thus the orientation of the hydrophone group, when a binaural balance is obtained, determines the direction of the sound source.

Instead of rotating the line of receivers in order to determine the direction of zero phase difference, it is sometimes more convenient to

compensate for this difference by introducing artificially an increased path length into the side nearer the source. In this case the phase difference of the arrival of the wave at the various receivers, as measured by the amount of retardation necessary in one path to bring the wave trains in it into phase with the wave trains of the other path, enables the direction of the sound source to be determined by a simple calculation.

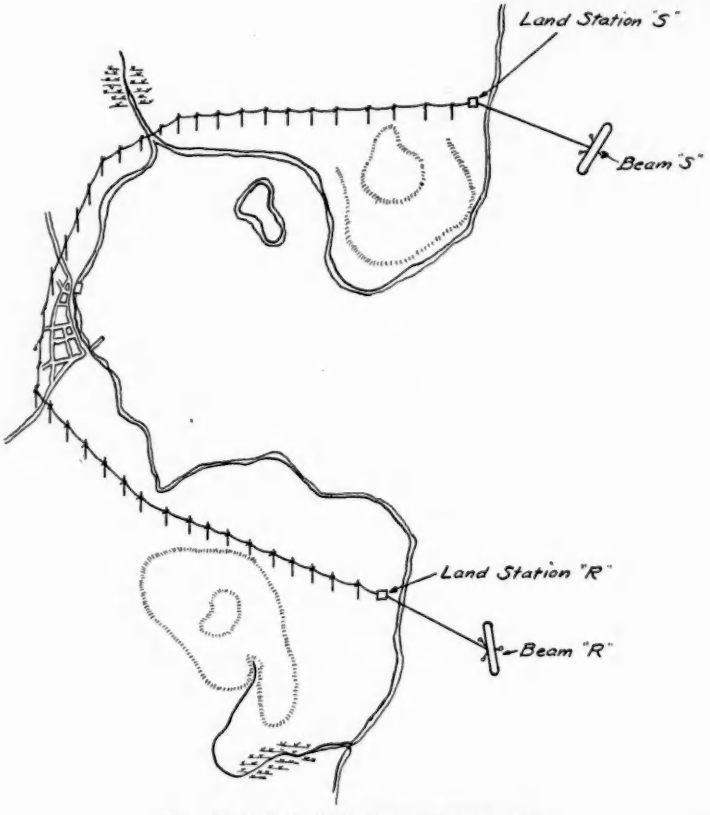

FIG. 7. *Stations protecting harbor entrance.*

The increase in path length between the hydrophones and the telephones receivers may be introduced electrically through the use of a properly constructed artificial telephone line whose equivalent length may be varied, or acoustically, through the use of air-tubes of adjustable length.

For position finding two groups of hydrophones separated by a considerable distance are used, as shown in Fig. 7. The intersection of two simultaneously measured bearings locates the position of the distant ship.

VELOCITY OF SOUND

While each of these methods of position finding possesses certain features which are not found in the other methods all have one point in common; all presume an accurate knowledge of the velocity of sound in sea water. It is not surprising to find, therefore, that the velocity of sound in water has been the subject of considerable study by the scientists and engineers of this and other countries. The velocity of sound in sea water depends upon the salinity, temperature and pressure of the water through which the sound passes and tables have been prepared giving the theoretical velocities for different values of these variables.

For survey work an accurate knowledge of the velocity of sound in water is of vital importance. Unfortunately, however, very little appears to be known about the path taken by the sound of a bomb explosion in reaching the shore stations, and it is, of course, difficult to determine the conditions affecting the velocity of sound throughout the area traversed by the sound waves. Furthermore, a question as to whether a higher velocity of the explosive wave near the actual explosion exists and introduces a serious error, has been brought up. The conclusions from investigations made by the British and American observers is that the distance through which the higher velocity exists is negligible, and that the wave travels with the velocity of sound almost immediately.

In view of these considerations it usually is considered advisable for survey work to measure the velocity of sound by actual tests on the working grounds whenever possible. For commercial purposes, where the extreme accuracy required for survey work is not necessary, the position can be found with sufficient accuracy through the use of an average value for the velocity of sound. The error introduced thereby is small and of no importance for this class of work.

COLLISION PREVENTION

The fact that none of these methods for position finding by underwater sound depends for its successful operation upon the conditions of visibility focuses attention on another important problem which confronts the mariner—the prevention of collision at sea. Modern underwater acoustics has demonstrated, already, its ability to fix the position of a ship at sea. Is it not possible that the problem of collision prevention can be solved by similar means? The value of any method for avoid-

ing collision depends, of course, to a considerable extent, upon the universality with which it is taken into use; for a ship which is unprovided with such a protective system becomes to a certain extent a hindrance to those who possess it. A number of suggestions involving the use of under-water sounds ignals for the prevention of collision have been made. Thus far none have received anything approaching universal recognition. Bearing in mind, however, the extent to which under-water sound signals are used for position finding, is it too much to expect that underwater acoustics may solve eventually the problem of collision prevention?

THE THEORY OF ACOUSTIC FILTRATION IN SOLID RODS

By R. B. LINDSAY AND F. E. WHITE
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ABSTRACT

The transmission of longitudinal waves through an infinitely long solid rod loaded at equal intervals with equal heavy masses is treated by a method analogous to the so-called "branch transmission" theory of acoustic filtration in air. The masses are assumed to be so concentrated that they move as a whole and longitudinal motions alone are considered. Transmission bands are found for frequencies satisfying the inequality relation

$$-1 \leqq \cos 2kl - \omega m/2\rho_0 cS \cdot \sin 2kl \leqq +1, \text{ where } k = \omega/c = 2\pi\nu/c$$

ν being the frequency and c the velocity of sound in the rod, m the mass of the concentrated load, S the cross-sectional area of the rod, ρ_0 the density of the rod material and $2l$ the distance between consecutive loads. The loaded rod is thus found to act like a low-pass filter with the cut-off frequency given by the first root of the transcendental equation $\omega m/2\rho_0 cS = \cot kl$. As the frequency increases there are alternate transmission and attenuation bands. Comparison is made between the theory and the experimental results of H. F. Olson (1925) and rather good agreement is found. The case of the transverse waves in a stretched string loaded at equal intervals with mass particles is also treated by the same method and the filter characteristics of this structure very simply obtained in agreement with the previous results of Crandall and others using different methods. Investigation is made of the propagation of longitudinal waves through a solid rod to which are attached at equal intervals strings loaded with masses at their free ends. It is shown that in general such a structure acts like a low-pass filter but that in the limiting case where the loaded ends are fastened to a completely rigid, motionless support the structure has the characteristics of a high-pass filter. Unfortunately the transmission and attenuation bands are of such a nature as to render such an arrangement of doubtful practical value.

INTRODUCTION

Some ten years ago, G. W. Stewart developed a theory for the filtration of sound in air,[1] and constructed acoustic filters with accurately predictable and relatively sharp cut-off frequencies. It was found, for example, that a tube with side branch attachments in the form of acoustic resonators regularly spaced behaves like a low-pass filter, while if the branch attachments are simple orifices, the filter is of the high-pass type. Stewart also envisaged the possibility of constructing solid filter structures. He and his students built and experimentally tested several of these.[2] Thus he found that a metal rod loaded at equal intervals with heavy masses (attached as collars to the rod) acts like a low-

[1] G. W. Stewart, Phys. Rev. **20**, 528 (1922); also Stewart and Lindsay, *Acoustics*, p. 159 ff.

[2] See, for example, *Acoustics*, p. 186 with the references there cited. Also V. C. Hall, Phys. Rev. **23**, 116A (1924).

pass filter to the passage of longitudinal waves through the rod. With the help of the lumped impedance theory which he had developed for filtration in air, he was able to account in a semi-empirical fashion for the main experimental results. But a more exact theoretical treatment promised to be rather difficult and was not undertaken. It is the purpose of the present paper to sketch a theoretical discussion of filtration in solids which may be reasonably compared with the available experimental data, and to suggest further possibilities for new structures.

THEORY OF THE PROPAGATION OF LONGITUDINAL WAVES THROUGH A LOADED ROD

We shall consider the propagation of dilatational waves through a solid rod loaded at equal intervals with heavy masses as indicated schematically in Fig. 1. The material of which the rod is made has mean

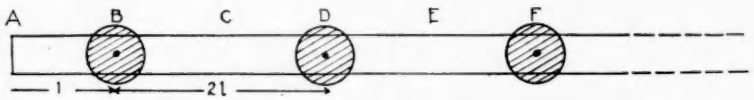

FIG. 1. *Schematic diagram of a loaded rod.*

equilibrium density ρ_0 and each load is of mass m. It will be assumed that the structure extends to infinity on the right. The method to be used envisages the effect of each load as that of a branch line on the propagation through the rod. This is effectively the method first proposed by Mason[3] for the treatment of the problem of filtration in air and presented in a somewhat modified form by Stewart and Lindsay[4] under the name "branch transmission theory." The longitudinal displacement in the rod due to a compressional disturbance ξ, satisfies the wave equation, viz.,

$$\partial^2\xi/\partial x^2 = \ddot{\xi}/c^2 \tag{1}$$

where the velocity of propagation, $c=(Y/\rho_0)^{1/2}$, and Y is Young's modulus. The solution of (1) for the special case of a harmonic wave if the x-axis is taken along the rod may be written in the form

$$\xi = \xi_0 e^{i(\omega t - kx)} + \xi_1 e^{i(\omega t + kx)} \tag{2}$$

where $\omega=2\pi\nu$, with ν the frequency of the wave, and $k=\omega/c=2\pi/\lambda$, if λ = wave-length. The two parts of the solution correspond to waves progressing in the positive and negative directions, respectively. ξ_0 and

[3] W. P. Mason, Bell System Tech. Jour. **6**, 258 (1927).
[4] *Acoustics*, p. 334.

ξ_1 are arbitrary amplitude constants to be determined in terms of the boundary conditions. The analogy with acoustic waves in a fluid is more emphatically brought out if we use the volume displacement $X=S\xi$ and the volume current $\dot{X}=S\dot{\xi}=i\omega S\xi$, where S is the area of cross section of the rod. Both X and $\dot{X}$, of course, satisfy the Eq. (1). Corresponding to the *excess pressure* in the case of a fluid and the electromotive force in the case of electric wave transmission we now have the *excess stress* or linear tension per unit area due to the passage of the dilatational wave through the rod. We shall denote this by T. From Hooke's law we have at once

$$\frac{T}{\partial\xi/\partial x} = Y = \rho_0 c^2. \tag{3}$$

It may be remarked that while acoustic impedance for waves in a fluid is defined by the ratio $p/\dot{X}$, p being the excess pressure, we may consistently define $-T/\dot{X}$ as the acoustic impedance of the wave in the rod. It can be easily shown that this leads to $\rho_0 c$ as the specific acoustic resistance for a plane progressive wave.[5]

From Eqs. (2) and (3) we obtain

$$\begin{aligned} \dot{X} &= i\omega S\left[\xi_0 e^{i(\omega t-kx)} + \xi_1 e^{i(\omega t+kx)}\right] \\ T &= i\omega\rho_0 c\left[-\xi_0 e^{i(\omega t-kx)} + \xi_1 e^{i(\omega t+kx)}\right]. \end{aligned} \tag{4}$$

If now we let the end of the rod denoted by A correspond to $x=0$ and suppose the excess stress and volume current there to be given by $T_1 e^{i\omega t}$ and $\dot{X}_1 e^{i\omega t}$, respectively, we can evaluate ξ_0 and ξ_1 in terms of T_1 and $\dot{X}_1$. Substitution into (4) with subsequent reduction then yields

$$\begin{aligned} \dot{X} &= [\dot{X}_1 \cos kx + iT_1/Z \cdot \sin kx]e^{i\omega t} \\ T &= [T_1 \cos kx + i\dot{X}_1 Z \cdot \sin kx]e^{i\omega t} \end{aligned} \tag{5}$$

wherein we have replaced $\rho_0 c/S$ by Z, the acoustic resistance for a plane progressive wave with wave front S.

The problem now is to compare $\dot{X}_{n+1}$ and $\dot{X}_n$, where $\dot{X}_n$ is the volume current half way between the $(n-1)$st and n-th loads. This will express the filtration characteristics of the structure. It will be assumed in our analysis that each load moves as a whole.

We have therefore to write the boundary conditions at each branch, expressing the continuity of displacement and stress. For this purpose we shall denote the displacement and stress in the rod immediately to

[5] See *Acoustics*, p. 106.

the left of the load at B by the subscript 12, while the subscript characteristic of these quantities in the rod immediately to the right of B will be 21. At the mid-branch points A, C, E, $\cdots$, the corresponding quantities will be characterized by the subscripts 1, 2, 3 $\cdots$. If now we denote the displacement of the load by ξ_L, the boundary conditions at B may be written

$$\dot{\xi}_{12} = \dot{\xi}_{21} = \dot{\xi}_L \tag{6}$$

$$S(T_{21} - T_{12}) = m\ddot{\xi}_L = i\omega m\dot{X}_L/S \tag{7}$$

where $\dot{X}_L = S\dot{\xi}_L$, to unify the notation. From Eqs. (5) if we leave off the time factor $e^{i\omega t}$ we have

$$\begin{aligned} \dot{X}_{12} &= \dot{X}_1 \cos kl + iT_1/Z \cdot \sin kl \\ T_{12} &= T_1 \cos kl + i\dot{X}_1 Z \cdot \sin kl. \end{aligned} \tag{8}$$

Likewise

$$\begin{aligned} \dot{X}_2 &= \dot{X}_{21} \cos kl + iT_{21}/Z \cdot \sin kl \\ T_2 &= T_{21} \cos kl + i\dot{X}_{21} Z \cdot \sin kl. \end{aligned} \tag{9}$$

Our task now is to apply the boundary conditions to Eqs. (8) and (9) so as to express $\dot{X}_2$ and T_2 in terms of $\dot{X}_1$ and T_1. On performing the necessary substitutions and reductions, we are finally led to

$$\begin{aligned} \dot{X}_2 = \dot{X}_1 &[\cos 2kl - \omega m/2ZS^2 \cdot \sin 2kl] \\ &+ iT_1/Z \cdot [\sin 2kl - \omega m/ZS^2 \cdot \sin^2 kl] \end{aligned} \tag{10}$$

$$\begin{aligned} T_2 = T_1 &[\cos 2kl - \omega m/2ZS^2 \cdot \sin 2kl] \\ &+ i\dot{X}_1 Z [\sin 2kl + \omega m/ZS^2 \cdot \cos^2 kl]. \end{aligned} \tag{11}$$

It has already been assumed that the structure is infinite toward the right. It therefore follows that the impedance is the same to the right of every section. Thus we write

$$-T_1/\dot{X}_1 = -T_2/\dot{X}_2 = \cdots = Z_0 \tag{12}$$

where the constant impedance has been represented by Z_0, usually referred to as the *characteristic* impedance. Substituting from (10) and (11) into (12) we solve for Z_0 and obtain

$$Z_0 = Z(1 + \omega m/2ZS^2 \cdot \cot kl)^{1/2}(1 - \omega m/2ZS^2 \cdot \tan kl)^{-1/2}. \tag{13}$$

It will now be convenient to let

$$\cos W = \cos 2kl - \omega m/2ZS^2 \cdot \sin 2kl, \tag{14}$$

whence it may be readily shown that

$$\begin{aligned} \sin W &= Z/Z_0 \cdot (\sin 2kl + \omega m/ZS^2 \cdot \cos^2 kl) \\ &= Z_0/Z \cdot (\sin 2kl - \omega m/ZS^2 \cdot \sin^2 kl). \end{aligned} \tag{15}$$

We may then write (10) and (11) in the form

$$\dot{X}_2 = \dot{X}_1 \cos W + iT_1/Z_0 \cdot \sin W \tag{16}$$

$$T_2 = T_1 \cos W + i\dot{X}_1 Z_0 \cdot \sin W. \tag{17}$$

These may at once be generalized to apply to any two successive sections such as the n-th and $(n+1)$st. Thus to concentrate on the volume current in particular

$$2\dot{X}_{n+1} = \dot{X}_n(e^{iW} + e^{-iW}) + T_n/Z_0(e^{iW} - e^{-iW}) \tag{18}$$

which since $Z_0 = -T_n/\dot{X}_n$, immediately reduces to the simple form

$$\dot{X}_{n+1} = \dot{X}_n e^{-iW}. \tag{19}$$

There is a similar relation between T_{n+1} and T_n. Inspection of (19) now shows that when W is *real*, the difference between $\dot{X}_{n+1}$ and $\dot{X}_n$ is one of phase only, i.e., there is no *attenuation* of the wave by the structure. On the other hand if W is complex, there will be attenuation whose magnitude will depend on the imaginary part of W. It is clear that for all frequency regions satisfying the inequality

$$+1 \geqq \cos W \geqq -1 \tag{20}$$

the structure will transmit longitudinal waves without attentuation (save that due to the ordinary dissipative processes which are neglected here). The upper and lower frequency limits of the transmission regions are given by setting cos W equal to the limiting values 1 and -1. We thus obtain, using Eq. (14) and replacing Z by $\rho_0 c/S$ the transcendental equations

$$\omega m/2\rho_0 cS = -\tan kl \tag{21}$$

$$\omega m/2\rho_0 cS = \cot kl. \tag{22}$$

Recalling that $k = \omega/c$, it is seen that Eq. (21) is satisfied by $\omega = 0$. Hence the structure under consideration is a *low-pass* filter, the first transmission region extending from $\omega = 0$ to the lowest value of ω satisfying Eq. (22). The latter gives the so-called *cut-off* frequency. The transmission characteristics of the system may perhaps be most clearly shown by plotting cos W directly as a function of frequency. Fig. 2 shows the plot of such a curve for the special case of a brass rod 0.32 cm² in cross-sectional area with $2l = 25.08$ cm and $m = 875$ grams. For brass $\rho_0 = 8.5$ grams/cm³ and $c = 3.5 \times 10^5$ cm/sec. The cross-hatched

regions are the transmission regions. It is seen that in this particular case we get transmission up to 1225 cycles before an attenuation region begins.

Before passing to a specific comparison of the theory with the available experimental data we may notice some general facts about the cut-off frequency given by (22). Writing the latter in the form

$$m/2\rho_0 Sl = \cot kl/kl \qquad (22')$$

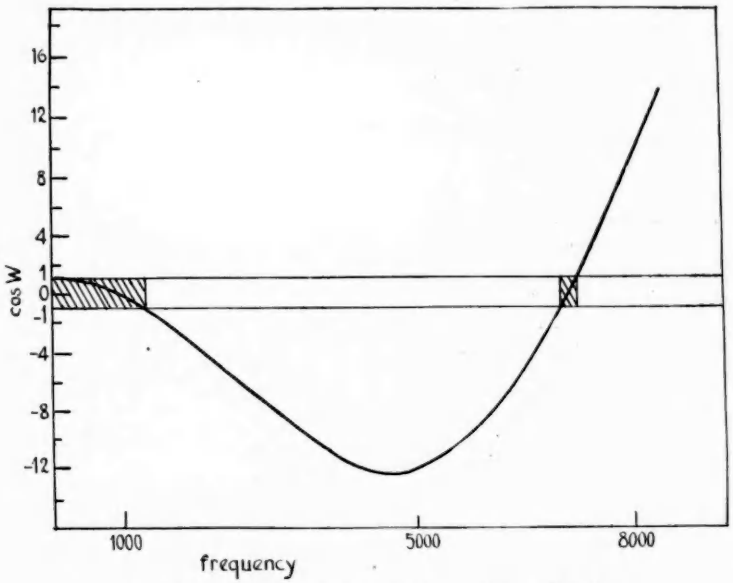

FIG. 2. *Transmission characteristic curve for a solid filter.*

we note that if for a given rod l is kept constant an *increase* in m, the mass of the load, *decreases* the cut-off frequency. Moreover if m is kept constant, we see that an *increase* in l also *decreases* the cut-off frequency.

H. F. Olson, one of G. W. Stewart's students, in 1925 constructed several solid filters and measured their cut-off frequencies.[6] The loads were in the form of cylindrical metal collars and the construction is indicated in the diagrams of Fig. 3 which show the two types used, viz., the O type with short necks and the N type with longer necks. We shall not give details about the methods of experimental measurement which were somewhat difficult to carry out, but shall merely compare some of the results with the corresponding theoretically calculated values. In

[6] So far as the authors are aware this work has not been published, the details being available in a master's thesis at the University of Iowa.

the case of each filter the cut-off frequency is indicated by plotting the transmission ratio P_r (=ratio of intensity after two or more sections to incident intensity) as a function of the frequency and noting at what frequency it becomes negligible. All the filters used were brass with $\rho_0 = 8.5$ g/cm³ and $c = 3.5 \times 10^5$ cm/sec. The dimensions given refer to Fig. 3 and all distances are in cm.

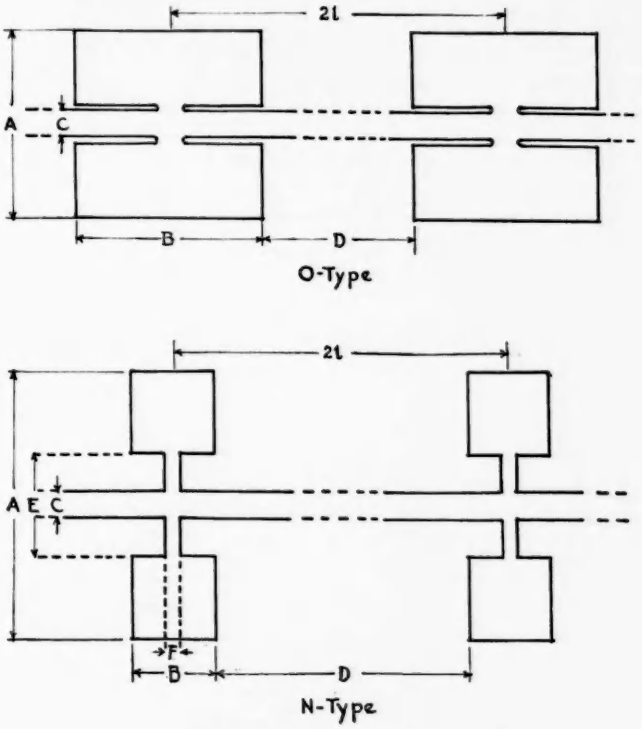

FIG. 3. *Diagrams of experimental solid filters.*

Filter 0–3–20

$A = B = 5.08$, $C = 0.64$, $D = 20.0$

$2l = 25.08$, $S = 0.32$ cm², $m = 875$ g

ν	500	600	700	800	1000	1100	1200	1400
P_r	0.20	0.29	0.28	0.35	0.22	0.22	0.045	0.0005

Cut-off observed 1200–1300 cycles
" " calculated 1225 "
(from Eq. 22)

Filter 0–3–40

Same as 0–3–20, save $D=40$, and $2l=45.08$

ν	500	600	700	800	900	1000	1100
P_r	0.27	0.42	0.29	0.26	.056	.017	.003

Cut-off observed 900–1000 cycles
" " calculated 910 "

Filter N_1–2–20

$A = 6.95$, $B = 2.15$, $C = 0.64$, $D = 20$

$E = 2.65$, $F = 0.45$, $2l = 22.15$

$S = 0.32$ cm², $m = 614$ gm

ν	500	600	800	1000	1200	1400	1600
P_r	0.28	0.22	0.35	0.32	0.20	0.0012	0.0008

Cut-off observed 1400 cycles (approx.)
" " calculated 1550 "

Filter N_3–2–20

$A = 8.50$, $B = 1.80$, $C = 0.64$, $D = 20$

$E = 4.90$, $F = 0.45$, $2l = 21.8$, $S = 0.32$ cm², $m = 652$ g

ν	500	600	800	900	1000	1100	1200
P_r	0.32	0.35	0.35	0.40	0.14	0.004	0.007

Cut-off observed 1100 cycles (approx.)
" " calculated 1510 "

In interpreting these results it must be borne in mind that the experimental results cannot hope to be known with extreme accuracy. The agreement between theory and experiment, therefore, especially in the case of the *O*-type structures, must be considered rather satisfactory. It will be noted that the only serious disagreement occurs for the case of an *N*-type structure. It is just here that we should expect the theory in its present form to be inadequate since it contemplates the motion of each load as a whole, while if the masses are relatively far from the rod, as in the *N*-type case, different portions of the load will move differently, i.e., there will be transverse as well as longitudinal waves set up in them. From the standpoint of the simple theory the general effect of this would be to make the effective mass of each load

larger than the actual mass. From what has been said above this would tend to lower the calculated cut-off frequencies and bring them more into line with the measured values. However, we do not wish to press these considerations for a more elaborate theory will be undertaken later. Nevertheless, it is felt that the simple theory here developed may prove to be of considerable utility.

THE LOADED STRING AS A TRANSVERSE WAVE FILTER

Before a further discussion of the filtration of longitudinal waves in solids, we may pause to recall that a stretched string loaded at equal intervals with mass particles acts as a low-pass filter for transverse waves. This has already been treated by Crandall[7] as a case of the forced oscillations of a system of many degrees of freedom. He used the method of Lagrange. It will, however, be of interest to apply the simpler method of the present paper to the same problem.

We shall assume that the string is infinite and stretched with mean tension τ_0. It is loaded with particles of mass m placed at equal intervals of length $2l$. If ξ now denotes the transverse displacement of the string, this will be propagated according to Eq. (1) with velocity $c=(\tau_0/\rho_0)^{1/2}$ where ρ_0 is here the *line density* or mass per unit length. The solution for the case of harmonic waves with the equilibrium position of the string along the x axis is then given by Eq. (2). Corresponding to Eq. (3) we now have the equation relating the instantaneous *normal component* of the tension in the string to the displacement. If we denote the former by τ, we have

$$\tau = \tau_0(\partial\xi/\partial x), \tag{23}$$

the mathematical connection with (3) being sufficiently obvious. The mathematical development now follows closely that for the solid rod with τ taking the place of T and $\dot{\xi}$ used in place of $\dot{X}$, since there is now no particular significance to be attached to a volume current. We thus have in place of Eqs. (4)

$$\begin{aligned} \dot{\xi} &= i\omega[\xi_0 e^{i(\omega t-kx)} + \xi_1 e^{i(\omega t+kx)}] \\ \tau &= i\omega\rho_0 c[-\xi_0 e^{i(\omega t-kx)} + \xi_1 e^{i(\omega t+kx)}] \end{aligned} \tag{24}$$

where ξ_0 and ξ_1 are now *transverse* displacement amplitudes. With $\dot{\xi}=\dot{\xi}_1 e^{i\omega t}$ and $\tau=\tau_1 e^{i\omega t}$ at $x=0$ we have corresponding to (5)

$$\begin{aligned} \dot{\xi} &= [\dot{\xi}_1 \cos kx + i\tau_1/\rho_0 c \cdot \sin kx]e^{i\omega t} \\ \tau &= [\tau_1 \cos kx + i\rho_0 c \cdot \dot{\xi}_1 \sin kx]e^{i\omega t}. \end{aligned} \tag{25}$$

[7] See *Theory of Vibrating Systems and Sound*, p. 64.

Making use of the same notation with regard to subscripts as in the case of the rod, we may write the boundary conditions analogous to (6) and (7) as follows

$$\dot{\xi}_{12} = \dot{\xi}_{21} = \dot{\xi}_L \tag{26}$$

$$\tau_{21} - \tau_{12} = m\ddot{\xi}_L \tag{27}$$

where ξ_L is the vertical displacement of the loading particle. The rest of the analysis is so completely analogous to that for the solid rod that we may omit it, merely noting that corresponding to (12) we may define the characteristic impedance of the infinite loaded string by

$$-\tau_1/\dot{\xi}_1 = -\tau_2/\dot{\xi}_2 = \cdots = Z_0'. \tag{28}$$

Setting

$$\cos W = \cos 2kl - \omega mc/2\tau_0 \cdot \sin 2kl \tag{29}$$

we finally arrive at

$$\dot{\xi}_{n+1} = \dot{\xi}_n e^{-iW} \tag{30}$$

$$\tau_{n+1} = \tau_n e^{-iW}. \tag{31}$$

It thus appears that the infinite loaded string transmits only those frequencies for which

$$-1 \leqq \cos 2kl - \omega mc/2\tau_0 \cdot \sin 2kl \leqq +1. \tag{32}$$

The limiting frequencies of the transmission region are then given by setting cos W equal to $+1$ and -1, respectively. We get thus the two transcendental equations

$$\omega mc/2\tau_0 = -\tan kl \tag{33}$$

$$\omega mc/2\tau_0 = \cot kl. \tag{34}$$

Since Eq. (33) is satisfied by $\omega = 0$, the loaded string behaves like a low-pass filter. The cut-off frequency for the first transmission band is given by (34). Let us suppose that kl is small; then to a first approximation cot $kl = kl$, and (34) becomes

$$\nu = \omega/2\pi = 1/2\pi \cdot (2\tau_0/ml)^{1/2}. \tag{35}$$

Now the n characteristic frequencies of a finite string loaded with n particles of mass m separated by equal distances $2l$ are[8]

$$\nu_1 = \frac{1}{2\pi}\left(\frac{2\tau_0}{ml}\right)^{1/2} \sin\frac{\pi}{2(n+1)}, \cdots \nu_n = \frac{1}{2\pi}\left(\frac{2\tau_0}{ml}\right)^{1/2} \sin\frac{n\pi}{2(n+1)}. \tag{36}$$

[8] See, for example, Crandall, *Theory of Vibrating Systems and Sound*, p. 67.

Crandall[8] has shown that under forced oscillation the finite string cuts off all frequencies above $\nu = (1/2\pi)\ (2\tau_0/ml)^{1/2}$, which is the limit of ν_n as n increases indefinitely. It is seen that this frequency agrees precisely with the cut-off frequency (35) obtained by the transmission theory used in this paper.

ALTERNATIVE TYPE OF LOADING FOR A SOLID ROD

Let us return to the case of an infinitely long solid rod and now consider it to be loaded at equal intervals with equal flexible strings or wires to the other extremities of which are attached equal weights of mass m (Fig. 4). We wish to examine the influence of this loading on the

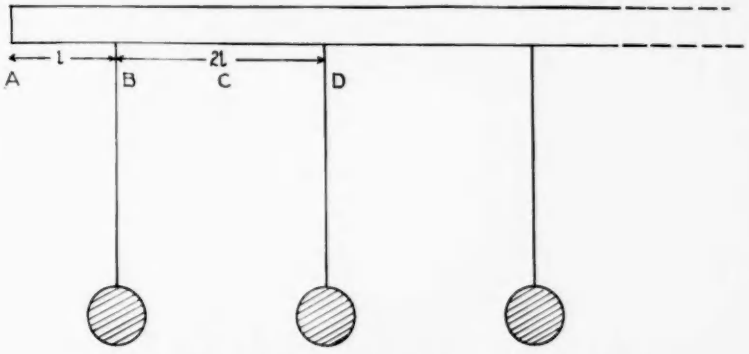

FIG. 4. *Schematic diagram of alternative type of loading of rod.*

propagation of longitudinal waves along the rod. The waves in the rod will act primarily to set up transverse waves in the wires which will then react on the rod at the junctions. This reaction will indeed have a component normal to the rod at each junction and hence flexural vibrations of the rod will necessarily be set up also. However, we shall assume that the rod is so supported that the amplitude of such vibrations is so small that their influence on the problem becomes negligible. In any case as a first approximation we shall consider only the longitudinal wave in the rod and the transverse wave in the strings.

Let us, as usual, consider that the x-axis lies along the rod and that the strings are parallel to the y-axis. The particle displacement for the disturbance in the rod is ξ, while the transverse displacement in the string is η. If the mean tension in each string is τ_0, the normal component, denoted by τ, will now be from (23), taking into account the necessary change in notation,

$$\tau = \tau_0(\partial\eta/\partial y), \tag{37}$$

The mechanical impedance for the transverse waves in the strings, which we shall here denote by Z_s, will then be

$$Z_s = -\tau/\dot{\eta}. \tag{38}$$

For the various quantities characterizing the longitudinal disturbance in the rod we choose the same convention with regard to subscripts as adopted in the first section of this paper. Hence for the boundary conditions at the junction B of the first string and the rod we shall have

$$\dot{\xi}_{12} = \dot{\eta}_1 = \dot{\xi}_{21} \tag{39}$$

$$ST_{12} - \tau_1 = ST_{21} \tag{40}$$

where $\dot{\eta}_1$ and τ_1 are respectively the transverse displacement and normal component of the tension of the string at the junction with the time factor $e^{i\omega t}$ omitted. S is as before, the area of cross section of the rod. The second of these conditions may be perhaps more conveniently written in terms of the impedance of the string at the junction, viz., $Z_{s1} = -\tau_1/\dot{\eta}_1$. Thus we have

$$ST_{12} + \dot{\eta}_1 Z_{s1} = ST_{21}. \tag{40a}$$

The procedure now follows exactly that in the first section (Eqs. (8)–(20) inclusive) and leads to the conclusion that the rod will transmit only those frequencies satisfying the inequality

$$-1 \leqq \cos W \leqq +1. \tag{41}$$

where

$$\cos W = \cos 2kl + iZ_{s1}/2ZS^2 \cdot \sin 2kl. \tag{42}$$

Here $Z = \rho_0 c/S$, as before (all quantities referring, of course, to the rod). Now in order to use the relation (41) to find the transmission bands we must be able to assign a value to Z_{s1}. This means that we must express the impedance at the junction end of the string in terms of that at the loaded end. For this purpose we may consult Eqs. (25) which give the transverse displacement velocity and normal tension component at any instant for any point of a transversely vibrating string in terms of the boundary values. Thus with appropriate change in notation we have

$$\begin{aligned} \dot{\eta} &= [\dot{\eta}_1 \cos k'y + i\tau_1/\rho_0' c' \cdot \sin k'y]e^{i\omega t} \\ \tau &= [\tau_1 \cos k'y + i\dot{\eta}_1' \rho_0' c' \sin k'y]e^{i\omega t}. \end{aligned} \tag{43}$$

The primed quantities refer to the string in order to avoid confusion with the corresponding quantities for the rod. Thus ρ_0' is the line density of the string, c' is the velocity of the transverse wave in the string

$(c' = (\tau_0/\rho_0')^{1/2})$ and $k' = \omega/c'$. Now if the length of each string is l', we shall have for the velocity and normal tension, respectively, at the loaded end

$$\begin{aligned} \dot{\eta}_{l'} &= [\dot{\eta}_1 \cos k'l' + i\tau_1/\rho_0'c' \cdot \sin k'l']e^{i\omega t} \\ \tau_{l'} &= [\tau_1 \cos k'l' + i\dot{\eta}_1\rho_0'c' \cdot \sin k'l']e^{i\omega t}. \end{aligned} \tag{44}$$

If $Z_{sl'}$ denotes the impedance of the string at the loaded end, we have from (44)

$$Z_{sl'} = -\frac{\tau_{l'}}{\dot{\eta}_{l'}} = -\frac{-Z_{s1}\cos k'l' + i\rho_0'c' \sin k'l'}{\cos k'l' - iZ_{s1}/\rho_0'c' \cdot \sin k'l'}. \tag{45}$$

Solving for Z_{sl} we obtain

$$Z_{s1} = \frac{Z_{sl'}\cos k'l' + i\rho_0'c' \sin k'l'}{\cos k'l' + iZ_{sl'}/\rho_0'c' \cdot \sin k'l'}. \tag{46}$$

In order to apply the method of the first section Z_{sl} must be a reactance (i.e., a pure imaginary impedance). This will be assured as long as $Z_{sl'}$ is a reactance. Neglecting dissipation through viscosity or radiation and recalling that the equation of motion of the load is

$$m\ddot{\eta}_{l'} = -\tau_{l'} \tag{47}$$

we have

$$Z_{sl'} = -\tau_{l'}/\dot{\eta}_{l'} = -i\omega\tau_{l'}/\ddot{\eta}_{l'} = i\omega m. \tag{48}$$

Hence

$$Z_{sl} = i\rho_0'c'(\omega m + \rho_0'c' \tan k'l')/(\rho_0'c' - \omega m \tan k'l'). \tag{49}$$

If we substitute this value of Z_{sl} into the inequality relation (41), the transmission bands are given by

$$\begin{aligned} -1 \leqq \cos 2kl - \rho_0'c'/2ZS^2 \\ (\omega m + \rho_0'c' \tan k'l')/(\rho_0'c' - \omega m \tan k'l') \cdot \sin 2kl \leqq +1. \end{aligned} \tag{50}$$

Now for finite m it is clear that when $\omega = 0$ the coefficient of $\sin 2kl$ is zero and hence $\cos W$ reduces to unity. Therefore in this case the structure behaves like a low-pass filter. The cut-off frequency is given by the transcendental equation (obtained by setting $\cos W = -1$)

$$\tan(\theta - k'l')/\tan kl = \rho_0'c'/2\rho_0cS \tag{51}$$

where $\theta = \text{arc tan}\,(\rho_0'c'/m\omega)$. If the angles are all small, as they will be for most practicable data, we may closely approximate the above equation by setting $\tan(\theta - k'l') = \theta - k'l'$ and $\tan kl = kl$. In this way the cut-off frequency is given by

$$\omega = \{\rho_0'c'^2/m[l' + l\rho_0'c'^2/2\rho_0c^2S]\}^{1/2}$$

which may be still further simplified by recalling that $c' = (\tau_0/\rho_0')^{1/2}$ and $\tau_0 = mg$. Hence

$$\omega = \{g/[l' + l\rho_0' c'^2/2\rho_0 c^2 S]\}^{1/2}. \tag{52}$$

It is clear that for practicable data the cut-off frequency will be comparatively low. Moreover inspection of (50) shows that the first attenuation band will ordinarily be very narrow and that except for this there will be transmission up to frequencies of several thousand cycles. It is thus seen that this type of structure is of doubtful value as a filter for longitudinal waves in solids.

There remains to be considered the theoretically interesting case in which $Z_{Sl'} = \infty$. Physically this corresponds to the attachment of the strings to a perfectly rigid motionless support. It is equivalent in our analysis to the assumption $m = \infty$ and with this hypothesis the transmission bands are given by the relation

$$-1 \leqq \cos 2kl + \rho_0' c'/2ZS^2 \cdot \cot k'l' \cdot \sin 2kl \leqq +1. \tag{53}$$

Now

$$\lim \cot k'l' \cdot \sin 2kl = 2lc'/cl' \tag{54}$$

and hence it is clear that for $\omega = 0$, $|\cos W| > 1$. Therefore, low frequencies are not transmitted and the structure behaves like a high-pass filter. The lower limiting frequency of the first transmission band is given by the equation

$$\rho_0' c'/2\rho_0 cS \cdot \cot k'l' = \tan kl. \tag{55}$$

Inspection of this equation shows that for reasonable physical data, the limiting frequency will be very small unless the ratio l/l' is very large. The practical significance of this arrangement is, consequently, probably rather slight, although it is felt that the same method applied to the case where the strings are replaced by suitably loaded *rods* may yield results of some interest.

BOOK REVIEW

Architectural Acoustics. VERN O. KNUDSEN. John Wiley and Sons, New York, 1932, pp. 617.

This book gives a refreshing account of the whole subject of architectural acoustics, such as might be expected from the enthusiasm and tireless energy of the author. It is not merely a compilation of standard facts and the substance of published articles, but it is largely an original discussion based on Dr. Knudsen's wide knowledge of acoustics and the direct information he has gained in active investigations. His earlier work in the design and operation of radio apparatus for the generation and reception of sound has been particularly helpful to him in obtaining experimental data on sound-absorption, sound-transmission, analysis of sound and the action of hearing.

There are three main divisions in the book,—the first part dealing with the elementary facts of physical and physiological acoustics; part two presenting the basic principles and data for the acoustical design of buildings, and including the novel features of the book with results of investigations hitherto unpublished by the author and with the most extensive tables of information about sound-absorptive and sound-insulative materials that the reviewer has seen anywhere; part three completing the book with a discussion of the practical acoustical design of various types of rooms and buildings. A large number of problems are given for teachers who plan to use the book for instructional purposes.

On reading the book, one is impressed with the thoroughness of treatment and the wealth of detail. While a brief review does not give opportunity for discussion of all the important features, attention may be called to Dr. Knudsen's extensive use of his "Percentage Articulation" (P. A.), which gives the percentage of typical speech sounds which are heard correctly by auditors. In equation form, this is written: P. A.$=96\ k_1k_2k_3k_4$, where the k's refer to the various factors that decrease correct hearing, namely, the shape of the room, inadequate loudness, excessive reverberation and extraneous noise. If the k's are all unity, the percentage articulation is 96 percent, which is the highest to be expected under perfect conditions; if the k's have values that reduce the percentage to 75 or less, the hearing conditions are unsatisfactory. Dr. Knudsen makes ingenious use of this relation to draw conclusions about optimal reverberation in rooms, the necessary loudness of a speaker's voice, the distance a speaker can be heard, etc., and generally he uses it to estimate the acoustical possibilities for speaking (and possibly music) under various conditions.

Numerical solutions are given for the acoustic adjustment of rooms and sound-proof partitions, with the commendable intention of aiding architects and others, but, except for simple problems, there are so many factors affecting the solution that an acoustical engineer really is needed who is familiar with the theory and practice. In other words, the subject of architectural acoustics has developed to such an extent that acoustical engineers are as necessary as engineers for the solution of the problems of lighting, heating and ventilation.

Dr. Knudsen's book is a valuable addition to the various publications already in the field, and should prove helpful in promoting acoustical excellence in buildings and in advancing the solutions of problems now under investigation.

F. R. WATSON